

GOVERNMENT OF KARNATAKA KARNATAKA STATE PRE-UNIVERSITY EDUCATION EXAMINATION BOARD II YEAR PUC EXAMINATION MARCH – APRIL 2012 SCHEME OF VALUATION		
Subject : MATHEMATICS		Subject Code : 35
Qn.No.		Marks Allotted
PART – A		
1	Write the prime power factorization of the composite number 45.	
Ans :	$45 = 3^2 \times 5^1$	1
2	Define a diagonal matrix.	
Ans :	Definition : A square matrix in which all the elements except the principal diagonal elements are zero is called a diagonal matrix. [Note : The diagonal elements may or may not be zero]	1
3	Why the usual division is not a binary operation on the set of real numbers ?	
Ans :	Since $\forall a, b, \in \mathbb{R}$, $\frac{a}{b} \notin \mathbb{R}$ when $b=0$	1
OR		
	Any counter example like $3, 0 \in \mathbb{R}$ but $\frac{3}{0}$ is not defined.	
4	Simplify $(\hat{i} \times \hat{j}) \times \hat{k}$	
Ans :	$(\hat{i} \times \hat{j}) \times \hat{k} = \hat{k} \times \hat{k} = \vec{0}$	1
	(Vector notation for zero vector is compulsory)	

Qn.No.		Marks Allotted
5	Find the centre of the circle $3x^2 + 3y^2 - 6x + 3y - 4 = 0$	
Ans :	Writing the equation as, $x^2 + y^2 - 2x + y - \frac{4}{3} = 0$ \therefore Centre is $\left(1, -\frac{1}{2}\right)$	1
6	For the parabola $2y^2 = 5x$ what is the length of the latus rectum?	
Ans :	Writing the equation as $y^2 = \frac{5}{2}x$ \therefore Length of LR = $4a = \frac{5}{2}$	1
7	Evaluate $\sin\left[\frac{1}{2}\cos^{-1}\left(\frac{4}{5}\right)\right]$	
Ans :	Let $\cos^{-1}\left(\frac{4}{5}\right) = \theta \quad \therefore \cos\theta = \frac{4}{5}$ $\therefore \sin\left(\frac{\theta}{2}\right) = \sqrt{\frac{1 - \cos\theta}{2}} = \sqrt{\frac{1 - \frac{4}{5}}{2}} = \frac{1}{\sqrt{10}}$	1
8	Express it in the polar form $z = 2 + i(2)$	
Ans :	$r = 2\sqrt{2}$, $\theta = \frac{\pi}{4}$ \therefore Polar form is $z = 2\sqrt{2} \operatorname{cis}\left(\frac{\pi}{4}\right)$	1

Qn.No.		Marks Allotted
9	If $y = \sin^{-1} x + \cos^{-1} x$ find $\frac{dy}{dx}$	
Ans:	$y = \frac{\pi}{2} \Rightarrow \frac{dy}{dx} = 0$	1
OR		
$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} + \left(\frac{-1}{\sqrt{1-x^2}} \right) = 0$		
10	Evaluate : $\int_{-\pi/2}^{\pi/2} (x + \sin x) dx$	
Ans:	Since the integrand is an odd function by a property $\int_{-\pi/2}^{\pi/2} (x + \sin x) dx = 0$	1
OR		
$\int_{-\pi/2}^{\pi/2} (x + \sin x) dx = \left[\frac{x^2}{2} - \cos x \right]_{-\pi/2}^{\pi/2} = 0$		
PART -B		
11	If $(a, b) = 1$, $(a, c) = 1$ then prove that $(a, bc) = 1$	
Ans:	$(a, b) = 1 \Rightarrow 1 = ax_1 + by_1$ and $(a, c) = 1 \Rightarrow 1 = ax_2 + cy_2$ Where $x_1, y_1, x_2, y_2 \in I$ (If one of the above equations is correct give one mark)	1
$\begin{aligned} \therefore 1 &= (ax_1 + by_1)(ax_2 + cy_2) \\ 1 &= a(ax_1x_2 + cx_1y_2 + bx_1y_1) + bc(y_1y_2) \\ 1 &= ax^1 + bcy^1 \quad \text{where } x^1, y^1 \in I \\ \Rightarrow (a, bc) &= 1 \end{aligned}$		

Qn.No.		Marks Allotted
12	Solve by Cramer's rule : $3x - y = 4$ $5x + 4y = 1$	
Ans :	$\Delta = 17$, $\Delta_1 = 17$ $\therefore x = 1$	1
	$\Delta_2 = -17$ $\therefore y = -1$	1
13	Prove that the identity element of a group $(G, *)$ is unique.	
Ans :	If e and e^1 are two different identity elements	
	Then $e * e^1 = e^1 * e = e^1$ and $e^1 * e = e * e^1 = e$ } $\Rightarrow e = e^1 \therefore$ Identity element is unique	1
14	Find the area of parallelogram whose diagonals are $3\hat{i} + \hat{j} - 2\hat{k}$ and $\hat{i} - 2\hat{j} + \hat{k}$	
Ans :	Let $\vec{d}_1 = (3, 1, -2)$ and $\vec{d}_2 = (1, -2, 1)$	
	$\therefore \vec{d}_1 \times \vec{d}_2 = -3\hat{i} - 5\hat{j} - 7\hat{k}$	1
	\therefore Area of the parallelogram = $\frac{1}{2} \vec{d}_1 \times \vec{d}_2 = \frac{\sqrt{83}}{2}$ sq. units	1
15	$3x + 4y = 2$ and $x - y = 3$ are equations of two diameters of a circle whose radius is 5 units. Find the equation of the circle.	
Ans :	Finding the centre = $(2, -1)$	1
	Writing the equation $(x - 2)^2 + (y + 1)^2 = 25$	1

Qn.No.		Marks Allotted
16	Find the centre of the ellipse $25x^2 + 16y^2 - 100x + 32y - 284 = 0$	
Ans :	Reducing to the standard form $\frac{(x-2)^2}{16} + \frac{(y+1)^2}{25} = 1$	1
	\therefore centre = (2, -1)	1
17	If $\tan^{-1} x + \tan^{-1} y = \frac{\pi}{4}$ then show that $x + y + xy = 1$	
Ans :	$\tan^{-1} \left(\frac{x+y}{1-xy} \right) = \frac{\pi}{4}$	1
	$\therefore \frac{x+y}{1-xy} = \tan \frac{\pi}{4} = 1 \Rightarrow x + y + xy = 1$	1
18	If $x + \frac{1}{x} = 2 \cos \theta$ then show that one of the values of x is $e^{i\theta}$	
Ans :	Getting $x^2 - 2x \cos \theta + 1 = 0$	1
	$\therefore x = \frac{2 \cos \theta \pm 2\sqrt{\cos^2 \theta - 1}}{2} = \cos \theta \pm i \sin \theta, e^{i\theta}$	1
19	If $y = \sqrt{\sin 2x + \sqrt{\sin 2x + \sqrt{\sin 2x + \dots}}}$ upto ∞ then show that $\frac{dy}{dx} = \frac{2 \cos 2x}{2y-1}$	
Ans:	Getting $y = \sqrt{\sin 2x + y}$	1
	Squaring and differentiating and getting $\frac{dy}{dx} = \frac{2 \cos 2x}{(2y-1)}$	1

Qn.No.		Marks Allotted
20	Find the y-intercept of the tangent drawn to the curve $y = 3x^2 - 4x$ at $(1, -1)$ on it	
Ans:	Getting $m = \left(\frac{dy}{dx}\right)_{(1,-1)} = 2$	1
	Equation of the tangent is $y = 2x - 3$ \therefore the y-intercept of the tangent is -3	1
21	Evaluate : $\int \frac{\cos x dx}{\sqrt{8 + \cos^2 x}}$	
Ans :	$\int \frac{\cos x}{\sqrt{8 + \cos^2 x}} dx = \int \frac{\cos x dx}{\sqrt{9 - \sin^2 x}}$	1
	Put $\sin x = t \Rightarrow \cos x dx = dt$ \therefore G.I = $\int \frac{dt}{\sqrt{9 - t^2}} = \sin^{-1}\left(\frac{\sin x}{3}\right) + C$	1
22	Find the order and degree of the differential equation $a^2 \frac{d^2y}{dx^2} = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{4}}$	
Ans:	Order of D.E = 2	1
	Degree of D.E = 4	1
I	PART - C	
23	Find the G.C.D of 111 and 409 and express it in two ways in the form $111m + 409n$ where $m, n \in \mathbb{Z}$	
Ans:	Division	1
	Writing $(111, 409) = 1$	1
	$1 = 409(19) + 111(-70)$ OR $111(-70) + 409(19)$	

Qn.No.		Marks Allotted
	$\therefore m = -70, n = 19$	1
	$1 = 111(-70) + 409(19) + 111 \times 409 - 111 \times 409$	1
	getting $m = 339, n = -92$	1
	OR	
	$m = -479, n = 130$	1
24.	Solve by matrix method $x+2y+3z=10, 2x-3y+z=1$ and $3x+y-2z=9$	
Ans:	Writing $X = A^{-1}B$ and $A^{-1} = \frac{\text{Adj}A}{ A }$	1
	$ A = 52$	1
	Cofactor matrix = $\begin{bmatrix} 5 & 7 & 11 \\ 7 & -11 & 5 \\ 11 & 5 & -7 \end{bmatrix}$	1
	(any four correct co-factors give one mark)	
	Adjoint of A = $\begin{bmatrix} 5 & 7 & 11 \\ 7 & -11 & 5 \\ 11 & 5 & -7 \end{bmatrix}$	1
	Getting $x = 3, y = 2$ and $z = 1$	1
25	Show that the set $G = \left\{ \frac{2^n}{n \in \mathbb{Z}} \right\}$ forms an abelian group w.r.t usual multiplication.	
Ans:	Note : Particular examples do not carry any mark	
	Closure law	1
	Associative law	1

Qn.No.		Marks Allotted
	$2^0 = 1$, is the identity element	1
	$\forall 2^n \in G, (2^n)^{-1} = 2^{-n} \in G$	1
	Commutative law	1
26 (a)	Prove that $[\bar{a} + \bar{b} \quad \bar{b} + \bar{c} \quad \bar{c} + \bar{a}] = 2[\bar{a} \quad \bar{b} \quad \bar{c}]$	
Ans:	Writing L.H.S = $(\bar{a} + \bar{b}) \cdot [(\bar{b} + \bar{c}) \times (\bar{c} + \bar{a})]$	1
	$= (\bar{a} + \bar{b}) \cdot [\bar{b} \times \bar{c} + \bar{b} \times \bar{a} + \bar{c} \times \bar{c} + \bar{c} \times \bar{a}]$	1
	$= \bar{a} \cdot (\bar{b} \times \bar{c}) + \bar{a} \cdot (\bar{b} \times \bar{a}) + \bar{a} \cdot (\bar{c} \times \bar{a})$ $+ \bar{b} \cdot (\bar{b} \times \bar{c}) + \bar{b} \cdot (\bar{b} \times \bar{a}) + \bar{b} \cdot (\bar{c} \times \bar{a})$	
	$= \bar{a} \cdot (\bar{b} \times \bar{c}) + \bar{b} \cdot (\bar{c} \times \bar{a})$	
	$= 2[\bar{a} \quad \bar{b} \quad \bar{c}]$	1
	[Deduct one mark if \cdot and \times are not mentioned]	
26 (b)	θ being angle between $\vec{a} = \hat{i} + \hat{j} - 3\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$ find the value of $\cos \theta$	
Ans :	$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{ \vec{a} \vec{b} }$	1
	$= \frac{2+1+3}{\sqrt{11} \sqrt{6}} = \frac{\sqrt{6}}{\sqrt{11}}$	1
	(Without the formula if the substitution is correct give one mark)	
II		
27(a)	Find the equation of the tangent to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ at a point $P(x_1, y_1)$ on it.	
Ans :	Correct figure [No figure or wrong figure carries zero mark]	1
	Slope of the tangent at $(x_1, y_1) = -\left(\frac{x_1 + g}{y_1 + f}\right)$	1

Qn.No.		Marks Allotted
	Getting the equation, $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$	1
27 (b)	What is the value of k if the circles $x^2 + y^2 + kx + 4y + 2 = 0$ and $2x^2 + 2y^2 - 4x - 3y + k = 0$ cut each other orthogonally ?	
Ans:	Writing the condition $2g_1g_2 + 2f_1f_2 = c_1 + c_2$	1
	Getting $k = \frac{-10}{3}$	1
28 (a)	Find the equation of the ellipse in the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, if the distance between foci is 8 and the distance between the directrices is 32.	
Ans :	Writing $2ae = 8$ and $\frac{2a}{e} = 32$ OR $2be = 8$ & $\frac{2b}{e} = 32$	1
	Getting $a = 8$, $e = \frac{1}{2}$ and $b^2 = 48$ $b = 8$, $e = \frac{1}{2}$ and $a^2 = 48$	1
	Writing the equation $\frac{x^2}{64} + \frac{y^2}{48} = 1$ $\frac{x^2}{48} + \frac{y^2}{64} = 1$	1
28 (b)	Find a point on the parabola $y^2 = 2x$ whose focal distance is $\frac{5}{2}$	
	Focal distance = $ x + a = \frac{5}{2}$	1
	Getting $a = \frac{1}{2}$ and $x = 2$, $y = \pm 2$	
	\therefore The point is (2, 2) OR (2, -2)	1
	(any one point is sufficient)	

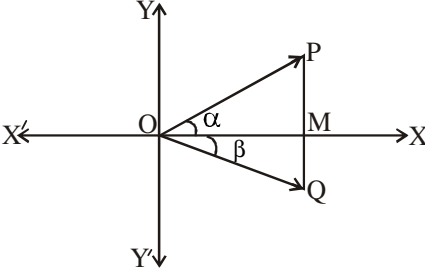
Qn.No.		Marks Allotted
29 (a)	Solve $\sin^{-1} x + \sin^{-1} 2x = \frac{\pi}{2}$	
Ans :	Let $\sin^{-1} x = A \Rightarrow \sin A = x \quad \therefore \cos A = \sqrt{1-x^2}$ $\sin^{-1} 2x = B \Rightarrow \sin B = 2x \quad \therefore \cos B = \sqrt{1-4x^2}$ and $A + B = \frac{\pi}{2}$	1
	Taking cos and getting $\sqrt{1-x^2} \sqrt{1-4x^2} - x(2x) = 0$	1
	Simplifying and getting $x = \frac{1}{\sqrt{5}}$	1
29 (b)	Find the general solution of $\tan \theta + \tan 2\theta + \sqrt{3} \tan \theta \tan 2\theta = \sqrt{3}$	
Ans :	Getting $\frac{\tan \theta + \tan 2\theta}{1 - \tan \theta \tan 2\theta} = \sqrt{3}$	1
	Writing $\tan 3\theta = \sqrt{3}$ and $\theta = \frac{n\pi}{3} + \frac{\pi}{9}, n \in I$	1
III		
30 (a)	Differentiate $\tan(ax)$ w.r.t x from first principles.	
Ans :	Writing $\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{\tan(ax + a\delta x) - \tan(ax)}{\delta x}$	1
	$= \lim_{\delta x \rightarrow 0} \frac{\sin(ax + a\delta x) \cos ax - \cos(ax + a\delta x) \sin ax}{\cos(ax + a\delta x) \cos ax}$	
	$= \lim_{\delta x \rightarrow 0} \frac{\sin a\delta x}{(\cos ax) \delta x \cdot \cos(ax + a\delta x)}$	1
	$\therefore \frac{dy}{dx} = a \sec^2 ax$	1
OR		

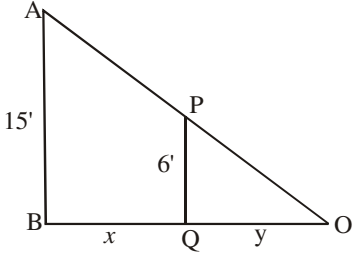
Qn.No.		Marks Allotted
	$\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{a \tan(a\delta x) [1 + \tan(ax + a\delta x) \tan ax]}{a \delta x}$	1
	$= a \sec^2 ax$	1
30 (b)	Differentiate w.r.t x : $(\cos x)^x$	
Ans :	Let $y = (\cos x)^x$	
	$\log y = x \log (\cos x)$	1
	Differentiating and getting $\frac{dy}{dx} = (\cos x)^x [-x \tan x + \log \cos x]$	1
31 (a)	If $y = e^{m \tan^{-1} x}$ then show that $(1+x^2)^2 y_2 + 2x(1+x^2) y_1 - m^2 y = 0$	
Ans :	$y_1 = m e^{m \tan^{-1} x} \cdot \frac{1}{1+x^2}$	1
	Again differentiating $(1+x^2) y_2 + y_1 2x = m y_1$	
	Substituting for y_1 in the R.H.S and getting $(1+x^2)^2 y_2 + 2x(1+x^2) y_1 - m^2 y = 0$	1
31 (b)	Differentiate $\cos(3x+2)$ w.r.t $3 \cos x$	
Ans :	Let $y = \cos(3x+2)$ and $z = 3 \cos x$	
	Getting $\frac{dy}{dx} = -\sin(3x+2) \cdot 3$ and $\frac{dz}{dx} = -3 \sin x$	1
	$\therefore \frac{dy}{dz} = \frac{\sin(3x+2)}{\sin x}$	1

Qn.No.		Marks Allotted
32 (a)	If $x\sqrt{1+y} + y\sqrt{1+x} = 0$ and $x \neq y$ then show that $\frac{dy}{dx} = \frac{-1}{(1+x)^2}$	
Ans :	$x\sqrt{1+y} = -y\sqrt{1+x}$	
	$x^2(1+y) = y^2(1+x)$	
	Getting $(x-y)[x+y+xy] = 0$	1
	Writing $y = \frac{-x}{1+x}$	1
	$\therefore \frac{dy}{dx} = \frac{-1}{(1+x)^2}$	1
	[for correct direct differentiation give one mark]	
32 (b)	Evaluate $\int \frac{(x-2)}{\sqrt{x+5}} dx$	
Ans :	Writing $I = \int \frac{x+5}{\sqrt{x+5}} dx - \int \frac{7}{\sqrt{x+5}} dx$	1
	$= \frac{2}{3} (x+5)^{\frac{3}{2}} - 14\sqrt{x+5} + C$	1
33 (a)	Evaluate $\int [e^x(1+x)\log_e(xe^x)] dx$	
Ans :	Put $xe^x = t$	1
	$\therefore I = \int \log t dt = t \log t - \int t \cdot \frac{1}{t} dt$	1
	$= t \log t - t + C$	

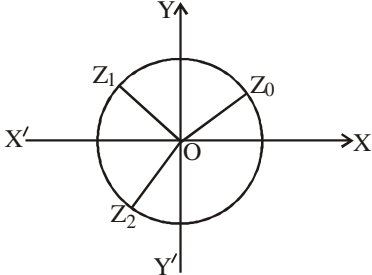
Qn.No.		Marks Allotted
33 (b)	Evaluate $\int (\tan x + \cot x)^2 dx$	
Ans :	$I = \int (\tan^2 x + \cot^2 x + 2) dx$	1
	$= \int (\sec^2 x + \operatorname{cosec}^2 x) dx = \tan x - \cot x + C$	1
	OR	
	$I = \int \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \right)^2 dx = \int \frac{4}{\sin^2 2x} dx$	
	$= 4 \int \operatorname{cosec}^2 2x dx$	1
	$= -2 \cot 2x + C$	1
34	Find the area of the circle $x^2 + y^2 = a^2$ by the method of integration	
Ans :	Correct figure [no figure, deduct one mark]	1
	Area of the circle $= 4 \int_0^a y dx = 4 \int_0^a \sqrt{a^2 - x^2} dx$	1
	$= 4 \left[\frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \right]_0^a$	1
	$= 4 \left[\frac{a}{2} (0) + \frac{a^2}{2} \sin^{-1}(1) \right] - 4[0 + 0]$	1
	$= \pi a^2 \text{ sq. units}$	1
	PART - D	
35 (a)	Define hyperbola as locus of a point and derive its equation in the standard form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	

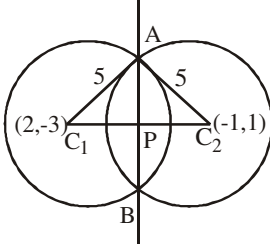
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Ans :	Definition as locus of a point only	1
	correct figure	1
	getting $cs = ae$	1
	and $cz = \frac{a}{e}$	1
	Writing $ps = epm$ ($e > 1$)	1
	Substituting for ps and pm and simplifying to get $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ where $b^2 = a^2(e^2 - 1)$ (No figure or wrong figure carries zero mark)	1
35 (b)	Prove that $\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2 \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$	
	By $c_1^1 = c_1 + c_2 - c_3$	
	L.H.S = $\begin{vmatrix} 2C & c+a & a+b \\ 2r & r+p & p+q \\ 2z & z+x & x+y \end{vmatrix}$	1
	= $2 \begin{vmatrix} c & c+a & a+b \\ r & r+p & p+q \\ z & z+x & x+y \end{vmatrix}$	1
	by $c_2^1 = c_2 - c_1$, $= 2 \begin{vmatrix} c & a & a+b \\ r & p & p+q \\ z & x & x+y \end{vmatrix}$ by $c_3^1 = c_3 - c_2$, $= 2 \begin{vmatrix} c & a & b \\ r & p & q \\ z & x & y \end{vmatrix}$	1

Qn.No.		Marks Allotted
	Interchanging c_1, c_2 and c_2, c_3	
	$= 2 \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$	1
36 (a)	State and prove the De Moivre's theorem for all rational indices	
Ans :	Statement : If n is an integer	
	$(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta$	
	and if n is a fraction then one of the values of $(\cos \theta + i \sin \theta)^n$ is $\cos n\theta + i \sin n\theta$	1
	Proving for positive integers	2
	Proving for negative integers	2
	Proving for fractions	1
36 (b)	Prove by vector method : $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$	
Ans :	<p>Let XOX' and YOY' be the co-ordinate axes</p> <p>Let $\overline{OP}, \overline{OQ}$ be two unit vectors with</p> <p>$X\hat{O}P = \alpha$ and</p> <p>$X\hat{O}Q = \beta$</p>	
		1
	Correct figure with explanation	1
	$\left. \begin{aligned} \overline{OP} &= \cos \alpha \hat{i} + \sin \alpha \hat{j} \\ \text{and } \overline{OQ} &= \cos \beta \hat{i} - \sin \beta \hat{j} \end{aligned} \right\}$	1
	$\overline{OP} \cdot \overline{OQ} = \overline{OP} \overline{OQ} \cos(\alpha + \beta)$	1
	Writing $\overline{OP} \cdot \overline{OQ} = \cos \alpha \cos \beta - \sin \alpha \sin \beta$	

Qn.No.		Marks Allotted
	and concluding $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$	1
37 (a)	A man 6 ft tall walks at a rate of 2ft / sec away from the source of light which is hung 15 ft above the horizontal ground. i) How fast is the length of his shadow increasing ? ii) How fast is the tip of his shadow moving ?	
Ans :	Figure [no figure carries zero mark]	1
		1
	From similar triangles, $\frac{AB}{PQ} = \frac{BO}{QO}$	1
	$\Rightarrow \frac{15}{6} = \frac{x+y}{y} \Rightarrow 2x = 3y$	1
	$\therefore 2 \frac{dx}{dt} = 3 \cdot \frac{dy}{dt} \Rightarrow \frac{dy}{dt} = \frac{4}{3}$ feet/sec	1
	Let $z = x + y$	1
	$\therefore \frac{dz}{dt} = \frac{dx}{dt} + \frac{dy}{dt} = \frac{10}{3}$ feet/sec	1
37 (b)	Find the general solution of $\cos \theta + \cos 3\theta + \cos 5\theta + \cos 7\theta = 0$	
Ans :	Writing $2 \cos 3\theta \cos 2\theta + 2 \cos 5\theta \cos 2\theta = 0$	1
	$2 \cos 2\theta [\cos 3\theta + \cos 5\theta] = 0$	
	$\Rightarrow 2 \cos 2\theta \cdot [2 \cos 4\theta \cdot \cos \theta] = 0$	
	$\Rightarrow \theta = n\pi \pm \frac{\pi}{4}$	1

Qn.No.		Marks Allotted
	$\theta = \frac{n\pi}{2} \pm \frac{\pi}{8}$ <p style="text-align: center;">OR</p> $\theta = (2n+1)\frac{\pi}{2}$ $\theta = (2n+1)\frac{\pi}{4}$	1
	$\theta = 2n\pi \pm \frac{\pi}{2}$ where n is an integer	1
38 (a)	Prove that $\int_0^1 \frac{\log_e(1+x)}{1+x^2} dx = \frac{\pi}{8} \log 2$	
Ans :	Put $x = \tan \theta$ and writing $I = \int_0^{\pi/4} \log(1 + \tan \theta) d\theta$	1
	Also $I = \int_0^{\pi/4} \log \left[1 + \tan \left(\frac{\pi}{4} - \theta \right) \right] d\theta$	1
	$= \int_0^{\pi/4} \log \left(\frac{2}{1 + \tan \theta} \right) d\theta$	1
	$= \int_0^{\pi/4} \log 2 d\theta - \int_0^{\pi/4} \log(1 + \tan \theta) d\theta$	1
	$\therefore 2I = \int_0^{\pi/4} \log 2 \cdot d\theta = \log 2 [\theta]_0^{\pi/4}$	1
	$\therefore I = \frac{\pi}{8} \log 2$	1
38 (b)	Find the general solution of $\frac{dy}{dx} = \sqrt{1-x^2-y^2+x^2y^2}$	
Ans :	$\frac{dy}{dx} = \sqrt{(1-x^2)(1-y^2)}$	1
	$\frac{dy}{\sqrt{1-y^2}} = \sqrt{1-x^2} dx$	1

Qn.No.		Marks Allotted
	$\int \frac{dy}{\sqrt{1-y^2}} = \int \sqrt{1-x^2} dx + C$	1
	$\sin^{-1}y = \frac{x}{2} \sqrt{1-x^2} + \frac{1}{2} \sin^{-1}x + C$	1
	(without C deduct one mark)	
PART - E		
39 (a)	Find the cube roots of $1+i\sqrt{3}$ and represent them in Argand diagram.	
Ans:	$r = 2 \quad \theta = \frac{\pi}{3}$	1
	$\therefore 1+i\sqrt{3} = 2 \operatorname{cis} \left[\frac{\pi}{3} \right]$	
	$= 2 \operatorname{cis} \left[2n\pi + \frac{\pi}{3} \right]$	1
	$= 2 \operatorname{cis} \left[\frac{6n\pi + \pi}{3} \right]$	
	Writing $z_0 = 2^{\frac{1}{3}} \operatorname{cis} \left(\frac{\pi}{9} \right)$, $z_1 = 2^{\frac{1}{3}} \operatorname{cis} \left(\frac{7\pi}{9} \right)$ and $z_2 = 2^{\frac{1}{3}} \operatorname{cis} \left(\frac{13\pi}{9} \right)$ }	1
	Argand diagram : 	1

Qn.No.		Marks Allotted
39 (b)	Find the length of the common chord of the intersecting circles $x^2 + y^2 - 4x + 6y - 12 = 0$ and $x^2 + y^2 + 2x - 2y - 23 = 0$	
Ans :	Figure 	1
	Getting radical axis $6x - 8y - 11 = 0$	1
	$C_1P = \frac{5}{2}$	1
	$AP = \sqrt{25 - \frac{25}{4}} = \frac{5\sqrt{3}}{2}$	
	\therefore length of the chord $AB = 5\sqrt{3}$ unit	1
39 (c)	Find the number of incongruent solutions and the incongruent solutions of linear congruence $5x \equiv 3 \pmod{6}$	
Ans :	$(5, 6) = 1 \quad \therefore$ no. of incongruent solution = 1	1
	$6 \mid 5x - 3 \quad \therefore x = 3$	1
40 (a)	If \vec{a}, \vec{b} are two unit vectors such that $(\vec{a} + \vec{b})$ is also a unit vector then show that angle between \vec{a} and \vec{b} is $\frac{2\pi}{3}$ and also show that $ \vec{a} - \vec{b} = \sqrt{3}$	
Ans :	Given $ \vec{a} + \vec{b} = 1 \Rightarrow \vec{a} + \vec{b} ^2 = 1$	
	$\Rightarrow (\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) = 1$	1
	Getting $\vec{a} \cdot \vec{b} = -\frac{1}{2}$ and $\theta = \frac{2\pi}{3}$	1

Qn.No.		Marks Allotted
	Writing $(\vec{a} - \vec{b}) \cdot (\vec{a} - \vec{b}) = \vec{a} ^2 + \vec{b} ^2 - 2\vec{a} \cdot \vec{b} = 3$	1
	Getting $ \vec{a} - \vec{b} ^2 = 3$ and $ \vec{a} - \vec{b} = \sqrt{3}$	1
40 (b)	Evaluate $\int \tan^4(3x) dx$	
Ans :	$I = \int \tan^2 3x \cdot \tan^2 3x dx$	1
	$= \int (\sec^2 3x - 1) \tan^2 3x dx$	1
	$= \int \sec^2 3x \tan^2 3x dx - \int (\sec^2 3x - 1) dx$	
	$= \frac{\tan^3 3x}{9} - \frac{\tan 3x}{3} + x + C$	1+1
	[If two integrals are correct give one mark]	
40 (c)	Differentiate w.r.t $x : \log_5(\log_e^x)$	
Ans :	Writing $y = \frac{\log(\log_e^x)}{\log 5}$	1
	$\therefore \frac{dy}{dx} = \frac{1}{\log 5} \cdot \frac{1}{\log x} \cdot \frac{1}{x}$	1