GAS POWER CYCLES

1.1 Theoretical Analysis

The accurate analysis of the various processes taking place in an internal combustion engine is a very complex problem. If these processes were to be analyzed experimentally, the analysis would be very realistic no doubt. It would also be quite accurate if the tests are carried out correctly and systematically, but it would be time consuming. If a detailed analysis has to be carried out involving changes in operating parameters, the cost of such an analysis would be quite high, even prohibitive. An obvious solution would be to look for a quicker and less expensive way of studying the engine performance characteristics. A theoretical analysis is the obvious answer.

A theoretical analysis, as the name suggests, involves analyzing the engine performance without actually building and physically testing an engine. It involves simulating an engine operation with the help of thermodynamics so as to formulate mathematical expressions which can then be solved in order to obtain the relevant information. The method of solution will depend upon the complexity of the formulation of the mathematical expressions which in turn will depend upon the assumptions that have been introduced in order to analyze the processes in the engine. The more the assumptions, the simpler will be the mathematical expressions and the easier the calculations, but the lesser will be the accuracy of the final results.

The simplest theoretical analysis involves the use of the air standard cycle, which has the largest number of simplifying assumptions.

1.2 A Thermodynamic Cycle

In some practical applications, notably steam power and refrigeration, a thermodynamic cycle can be identified.

A thermodynamic cycle occurs when the working fluid of a system experiences a number of processes that eventually return the fluid to its initial state.

In steam power plants, water is pumped (for which work $W_P$ is required) into a boiler and evaporated into steam while heat $Q_A$ is supplied at a high temperature. The steam flows...
through a turbine doing work $W_T$ and then passes into a condenser where it is condensed into water with consequent rejection of heat $Q_R$ to the atmosphere. Since the water is returned to its initial state, the net change in energy is zero, assuming no loss of water through leakage or evaporation.

An energy equation pertaining only to the system can be derived. Considering a system with one entering and one leaving flow stream for the time period $t_1$ to $t_2$

$$\Delta Q - \Delta W + \Delta E_{in} - \Delta E_{out} = \Delta E_{system} \quad (1)$$

$\Delta Q$ is the heat transfer across the boundary, $+ve$ for heat added to the system and $-ve$ for heat taken from the system.

$\Delta W$ is the work transfer across the boundary, $+ve$ for work done by the system and $-ve$ for work added to the system.

$\Delta E_{in}$ is the energy of all forms carried by the fluid across the boundary into the system.

$\Delta E_{out}$ is the energy of all forms carried by the fluid across the boundary out of system.

$\Delta E_{system}$ is the energy of all forms stored within the system, $+ve$ for energy increase $-ve$ for energy decrease.

In the case of the steam power system described above

$$Q_A + Q_R = \sum Q = \sum W = W_T + W_p \quad (2)$$

All thermodynamic cycles have a heat rejection process as an invariable characteristic and the net work done is always less than the heat supplied, although, as shown in Eq. 2, it is equal to the sum of heat added and the heat rejected ($Q_R$ is a negative number).

The thermal efficiency of a cycle, $\eta_{th}$, is defined as the fraction of heat supplied to a thermodynamic cycle that is converted to work, that is
\[ \eta_{th} = \frac{\sum W}{Q_A} \]

\[ = \frac{Q_A + Q_R}{Q_A} \]  \hspace{1cm} (3)

This efficiency is sometimes confused with the enthalpy efficiency, \( \eta_e \), or the fuel conversion efficiency, \( \eta_f \)

\[ \eta_e = \frac{\sum W}{m_f Q_c} \]  \hspace{1cm} (4)

This definition applies to combustion engines which have as a source of energy the chemical energy residing in a fuel used in the engine.

Any device that operated in a thermodynamic cycle, absorbs thermal energy from a source, rejects a part of it to a sink and presents the difference between the energy absorbed and energy rejected as work to the surroundings is called a heat engine.

A heat engine is, thus, a device that produces work. In order to achieve this purpose, the heat engine uses a certain working medium which undergoes the following processes:

1. A compression process where the working medium absorbs energy as work.
2. A heat addition process where the working medium absorbs energy as heat from a source.
3. An expansion process where the working medium transfers energy as work to the surroundings.
4. A heat rejection process where the working medium rejects energy as heat to a sink.

If the working medium does not undergo any change of phase during its passage through the cycle, the heat engine is said to operate in a non-phase change cycle. A phase change cycle is one in which the working medium undergoes changes of phase. The air standard cycles, using air as the working medium are examples of non-phase change cycles while the steam and vapor compression refrigeration cycles are examples of phase change cycles.
1.3 Air Standard Cycles

The air standard cycle is a cycle followed by a heat engine which uses air as the working medium. Since the air standard analysis is the simplest and most idealistic, such cycles are also called *ideal cycles* and the engine running on such cycles are called *ideal engines*.

In order that the analysis is made as simple as possible, certain assumptions have to be made. These assumptions result in an analysis that is far from correct for most actual combustion engine processes, but the analysis is of considerable value for indicating the upper limit of performance. The analysis is also a simple means for indicating the relative effects of principal variables of the cycle and the relative size of the apparatus.

**Assumptions**

1. The working medium is a perfect gas with constant specific heats and molecular weight corresponding to values at room temperature.
2. No chemical reactions occur during the cycle. The heat addition and heat rejection processes are merely heat transfer processes.
3. The processes are reversible.
4. Losses by heat transfer from the apparatus to the atmosphere are assumed to be zero in this analysis.
5. The working medium at the end of the process (cycle) is unchanged and is at the same condition as at the beginning of the process (cycle).

In selecting an idealized process one is always faced with the fact that the simpler the assumptions, the easier the analysis, but the farther the result from reality. The air cycle has the advantage of being based on a few simple assumptions and of lending itself to rapid and easy mathematical handling without recourse to thermodynamic charts or tables or complicated calculations. On the other hand, there is always the danger of losing sight of its limitations and of trying to employ it beyond its real usefulness.
Equivalent Air Cycle

A particular air cycle is usually taken to represent an approximation of some real set of processes which the user has in mind. Generally speaking, the air cycle representing a given real cycle is called an equivalent air cycle. The equivalent cycle has, in general, the following characteristics in common with the real cycle which it approximates:

1. A similar sequence of processes.
2. Same ratio of maximum to minimum volume for reciprocating engines or maximum to minimum pressure for gas turbine engines.
3. The same pressure and temperature at a given reference point.
4. An appropriate value of heat addition per unit mass of air.

1.4 The Carnot Cycle

This cycle was proposed by Sadi Carnot in 1824 and has the highest possible efficiency for any cycle. Figures 1 and 2 show the P-V and T-s diagrams of the cycle.

Fig.1: P-V Diagram of Carnot Cycle. Fig.2: T-S Diagram of Carnot Cycle.

Assuming that the charge is introduced into the engine at point 1, it undergoes isentropic compression from 4 to 1. The temperature of the charge rises from $T_{\text{min}}$ to $T_{\text{max}}$. At point 2, heat is added isothermally. This causes the air to expand, forcing the piston forward, thus doing work on the piston. At point 3, the source of heat is removed at constant temperature. At point 4, a cold body is applied to the end of the cylinder and the piston reverses, thus compressing the air isothermally; heat is rejected to the cold body. At point 1, the cold body is removed and the
charge is compressed isentropically till it reaches a temperature $T_{\text{max}}$ once again. Thus, the heat addition and rejection processes are isothermal while the compression and expansion processes are isentropic.

From thermodynamics, per unit mass of charge

\[
\text{Heat supplied from point 1 to 2} = p_2 v_2 \ln \frac{v_2}{v_1} \quad (5)
\]

\[
\text{Heat rejected from point 3 to 4} = p_4 v_3 \ln \frac{v_4}{v_3} \quad (6)
\]

Now $p_2 v_2 = R T_{\text{max}} \quad (7)$

And $p_4 v_4 = R T_{\text{min}} \quad (8)$

Since Work done, per unit mass of charge, $W = \text{heat supplied} - \text{heat rejected}$

\[
W = R T_{\text{max}} \ln \frac{v_3}{v_2} - R T_{\text{min}} \ln \frac{v_1}{v_4}
\]

\[
= R \ln (r) \left( T_{\text{max}} - T_{\text{min}} \right) \quad (9)
\]

We have assumed that the compression and expansion ratios are equal, that is

\[
\frac{v_3}{v_2} = \frac{v_1}{v_4} \quad (10)
\]

Heat supplied $Q_s = R T_{\text{max}} \ln (r) \quad (11)$

Hence, the thermal efficiency of the cycle is given by

\[
\eta_{th} = \frac{R \ln (r) \left( T_{\text{max}} - T_{\text{min}} \right)}{R \ln (r) T_{\text{max}}}
\]

\[
= \frac{T_{\text{max}} - T_{\text{min}}}{T_{\text{max}}} \quad (12)
\]

From Eq. 12 it is seen that the thermal efficiency of the Carnot cycle is only a function of the maximum and minimum temperatures of the cycle. The efficiency will increase if the minimum temperature (or the temperature at which the heat is rejected) is as low as possible.
According to this equation, the efficiency will be equal to 1 if the minimum temperature is zero, which happens to be the absolute zero temperature in the thermodynamic scale.

This equation also indicates that for optimum (Carnot) efficiency, the cycle (and hence the heat engine) must operate between the limits of the highest and lowest possible temperatures. In other words, the engine should take in all the heat at as high a temperature as possible and should reject the heat at as low a temperature as possible. For the first condition to be achieved, combustion (as applicable for a real engine using fuel to provide heat) should begin at the highest possible temperature, for then the irreversibility of the chemical reaction would be reduced. Moreover, in the cycle, the expansion should proceed to the lowest possible temperature in order to obtain the maximum amount of work. These conditions are the aims of all designers of modern heat engines. The conditions of heat rejection are governed, in practice, by the temperature of the atmosphere.

It is impossible to construct an engine which will work on the Carnot cycle. In such an engine, it would be necessary for the piston to move very slowly during the first part of the forward stroke so that it can follow an isothermal process. During the remainder of the forward stroke, the piston would need to move very quickly as it has to follow an isentropic process. This variation in the speed of the piston cannot be achieved in practice. Also, a very long piston stroke would produce only a small amount of work most of which would be absorbed by the friction of the moving parts of the engine.

Since the efficiency of the cycle, as given by Eq. 11, is dependent only on the maximum and minimum temperatures, it does not depend on the working medium. It is thus independent of the properties of the working medium.

1.5 The Otto Cycle

The Otto cycle, which was first proposed by a Frenchman, Beau de Rochas in 1862, was first used on an engine built by a German, Nicholas A. Otto, in 1876. The cycle is also called a constant volume or explosion cycle. This is the equivalent air cycle for reciprocating piston engines using spark ignition. Figures 5 and 6 show the P-V and T-s diagrams respectively.
At the start of the cycle, the cylinder contains a mass $M$ of air at the pressure and volume indicated at point 1. The piston is at its lowest position. It moves upward and the gas is compressed isentropically to point 2. At this point, heat is added at constant volume which raises the pressure to point 3. The high pressure charge now expands isentropically, pushing the piston down on its expansion stroke to point 4 where the charge rejects heat at constant volume to the initial state, point 1.

The isothermal heat addition and rejection of the Carnot cycle are replaced by the constant volume processes which are, theoretically more plausible, although in practice, even these processes are not practicable.

The heat supplied, $Q_s$, per unit mass of charge, is given by

$$c_v(T_3 - T_2)$$  \hspace{1cm} (13)

the heat rejected, $Q_r$ per unit mass of charge is given by

$$c_v(T_4 - T_1)$$  \hspace{1cm} (14)

and the thermal efficiency is given by
\[ \eta_{th} = 1 - \frac{(T_s - T_1)}{(T_3 - T_2)} \]

\[ = 1 - \frac{T_1}{T_2} \left\{ \frac{T_3 - 1}{T_1 - 1} \right\} \quad (15) \]

Now \[ \frac{T_1}{T_2} = \left( \frac{V_2}{V_1} \right)^{\gamma - 1} = \left( \frac{V_3}{V_4} \right)^{\gamma - 1} = \frac{T_4}{T_3} \]

And since \[ \frac{T_1}{T_2} = \frac{T_4}{T_3} \] we have \[ \frac{T_4}{T_1} = \frac{T_3}{T_2} \]

Hence, substituting in Eq. 15, we get, assuming that \( r \) is the compression ratio \( V_1/V_2 \)

\[ \eta_{th} = 1 - \frac{T_1}{T_2} \]

\[ = 1 - \left( \frac{V_2}{V_1} \right)^{\gamma - 1} \]

\[ = 1 - \frac{1}{r^{\gamma - 1}} \quad (16) \]

In a true thermodynamic cycle, the term \textit{expansion ratio} and \textit{compression ratio} are synonymous. However, in a real engine, these two ratios need not be equal because of the valve timing and therefore the term \textit{expansion ratio} is preferred sometimes.

Equation 16 shows that the thermal efficiency of the theoretical Otto cycle increases with increase in compression ratio and specific heat ratio but is independent of the heat added (independent of load) and initial conditions of pressure, volume and temperature.

Figure 5 shows a plot of thermal efficiency versus compression ratio for an Otto cycle. It is seen that the increase in efficiency is significant at lower compression ratios. This is also seen in Table 1 given below.
From the table it is seen that if:

- CR is increased from 2 to 4, efficiency increase is 76%.
- CR is increased from 4 to 8, efficiency increase is only 32.6%.
CR is increased from 8 to 16, efficiency increase is only 18.6%

**Mean effective pressure:**

It is seen that the air standard efficiency of the Otto cycle depends only on the compression ratio. However, the pressures and temperatures at the various points in the cycle and the net work done, all depend upon the initial pressure and temperature and the heat input from point 2 to point 3, besides the compression ratio.

A quantity of special interest in reciprocating engine analysis is the mean effective pressure. Mathematically, it is the net work done on the piston, \( W \), divided by the piston displacement volume, \( V_1 - V_2 \). This quantity has the units of pressure. Physically, it is that constant pressure which, if exerted on the piston for the whole outward stroke, would yield work equal to the work of the cycle. It is given by

\[
mep = \frac{W}{V_1 - V_2} = \eta \frac{Q_{2-3}}{V_1 - V_2} \tag{17}
\]

where \( Q_{2-3} \) is the heat added from points 2 to 3.

Work done per kg of air

\[
W = \frac{P_3V_3 - P_4V_4}{\nu - 1} - \frac{P_2V_2 - P_1V_1}{\nu - 1} = mepV_1 = P_m(V_1 - V_2)
\]

\[
mep = \frac{1}{(V_1 - V_2)} \left[ \frac{P_3V_3 - P_4V_4}{\nu - 1} - \frac{P_2V_2 - P_1V_1}{\nu - 1} \right] \tag{17A}
\]

The pressure ratio \( P_3/P_2 \) is known as explosion ratio \( r_p \)

\[
\frac{P_2}{P_1} = \left( \frac{V_1}{V_2} \right)^\nu = r^\nu \Rightarrow P_2 = P_1r^\nu,
\]

\[
P_3 = P_2r_p = P_1r^\nu r_p,
\]

\[
P_4 = P_3 \left( \frac{V_3}{V_4} \right)^\nu = P_1r^\nu r_p \left( \frac{V_2}{V_1} \right)^\nu = P_1r^\nu r_p
\]
\[
\frac{V_1}{V_2} = \frac{V_c + V_s}{V_c} = r \\
\therefore V_s = V_c(r - 1)
\]

Substituting the above values in Eq 17A

\[
\textit{mep} = P_1 \left( \frac{r(r_p - 1)(r_r - 1)}{(r - 1)(\gamma - 1)} \right)
\]

Now

\[
V_1 - V_2 = V_1 \left( 1 - \frac{V_2}{V_1} \right)
\]

\[
= V_1 \left( 1 - \frac{1}{r} \right) \quad (18)
\]

Here \( r \) is the compression ratio, \( V_1/V_2 \)

From the equation of state:

\[
V_1 = M \frac{R_0}{m} \frac{T_1}{P_1} \quad (19)
\]

\( R_0 \) is the universal gas constant

Substituting for \( V_1 \) from Eq. 3 in Eq. 2 and then substituting for \( V_1 - V_2 \) in Eq. 1 we get

\[
\textit{mep} = \eta \frac{Q_{2-3}}{MR_0 T_1} \frac{P_1 m}{1 - \frac{1}{r}} \quad (20)
\]

The quantity \( Q_{2-3}/M \) is the heat added between points 2 and 3 \textit{per unit mass} of air (\( M \) is the mass of air and \( m \) is the molecular weight of air); and is denoted by \( Q' \), thus

\[
\textit{mep} = \eta \frac{Q'}{R_0 T_1} \frac{P_1 m}{1 - \frac{1}{r}} \quad (21)
\]
We can non-dimensionalize the mep by dividing it by \( p_1 \) so that we can obtain the following equation

\[
\frac{mep}{p_1} = \eta \left[ \frac{1}{1 - \frac{1}{r}} \right] \left[ \frac{Q'm}{R_0 T_i} \right]
\] (22)

Since \( \frac{R_0}{m} = c_v(\gamma - 1) \), we can substitute it in Eq. 25 to get

\[
\frac{mep}{p_1} = \eta \frac{Q'}{c_v T_i} \left[ \frac{1}{1 - \frac{1}{r}} \right] \left[ \frac{1}{\gamma - 1} \right]
\] (23)

The dimensionless quantity \( \frac{mep}{p_1} \) is a function of the heat added, initial temperature, compression ratio and the properties of air, namely, \( c_v \) and \( \gamma \). We see that the mean effective pressure is directly proportional to the heat added and inversely proportional to the initial (or ambient) temperature.

We can substitute the value of \( \eta \) from Eq. 20 in Eq. 26 and obtain the value of \( \frac{mep}{p_1} \) for the Otto cycle in terms of the compression ratio and heat added.

In terms of the pressure ratio, \( p_3/p_2 \) denoted by \( r_p \) we could obtain the value of \( \frac{mep}{p_1} \) as follows:

\[
\frac{mep}{p_1} = \frac{r_p(r_p - 1)(r^{\gamma - 1} - 1)}{(r - 1)(\gamma - 1)}
\] (24)

We can obtain a value of \( r_p \) in terms of \( Q' \) as follows:

\[
r_p = \frac{Q'}{c_v T_i r^{\gamma - 1}} + 1
\] (25)

**Choice of \( Q' \)**

We have said that

\[
Q' = \frac{Q_{2-3}}{M}
\] (26)
M is the mass of charge (air) per cycle, kg.

Now, in an actual engine

\[ Q_{2-3} = M_f Q_c \]

\[ = F M_a Q_c \text{ in kJ / cycle} \quad (27) \]

\( M_f \) is the mass of fuel supplied per cycle, kg

\( Q_c \) is the heating value of the fuel, kJ/kg

\( M_a \) is the mass of air taken in per cycle

\( F \) is the fuel air ratio = \( M_f / M_a \)

Substituting for Eq. (B) in Eq. (A) we get

\[ Q' = \frac{F M_a Q_c}{M} \quad (28) \]

Now \( \frac{M_a}{M} \approx \frac{V_1 - V_2}{V_1} \)

And \( \frac{V_1 - V_2}{V_1} = 1 - \frac{1}{r} \quad (29) \)

So, substituting for \( M_a/M \) from Eq. (33) in Eq. (32) we get

\[ Q' = F Q_c \left( 1 - \frac{1}{r} \right) \quad (30) \]

For iso-octane, \( F Q_c \) at stoichiometric conditions is equal to 2975 kJ/kg, thus

\[ Q' = 2975(r - 1)/r \quad (31) \]

At an ambient temperature, \( T_1 \) of 300K and \( c_v \) for air is assumed to be 0.718 kJ/kgK, we get a value of \( Q'/c_vT_1 = 13.8(r - 1)/r \).

Under fuel rich conditions, \( \phi = 1.2 \), \( Q'/ c_vT_1 = 16.6(r - 1)/r \). \quad (32)

Under fuel lean conditions, \( \phi = 0.8 \), \( Q'/ c_vT_1 = 11.1(r - 1)/r \) \quad (33)
1.6 The Diesel Cycle

This cycle, proposed by a German engineer, Dr. Rudolph Diesel to describe the processes of his engine, is also called the constant pressure cycle. This is believed to be the equivalent air cycle for the reciprocating slow speed compression ignition engine. The P-V and T-s diagrams are shown in Figs 6 and 7 respectively.

**Fig.6: P-V Diagram of Diesel Cycle.**

**Fig.7: T-S Diagram of Diesel Cycle.**
The cycle has processes which are the same as that of the Otto cycle except that the heat is added at constant pressure.

The heat supplied, $Q_s$ is given by

$$c_p(T_3 - T_2)$$

(34)

whereas the heat rejected, $Q_r$ is given by

$$c_v(T_4 - T_1)$$

(35)

and the thermal efficiency is given by

$$\eta_{th} = 1 - \frac{c_v(T_4 - T_1)}{c_p(T_3 - T_2)}$$

$$= 1 - \frac{1}{\gamma} \left[ \frac{T_1}{T_3} \left( \frac{T_4}{T_1} - 1 \right) \right]$$

(36)

From the T-s diagram, Fig. 7, the difference in enthalpy between points 2 and 3 is the same as that between 4 and 1, thus

$$\Delta s_{2-3} = \Delta s_{4-1}$$

$$\therefore c_v \ln \left( \frac{T_4}{T_1} \right) = c_p \ln \left( \frac{T_3}{T_2} \right)$$

$$\therefore \ln \left( \frac{T_4}{T_1} \right) = \gamma \ln \left( \frac{T_3}{T_2} \right)$$

$$\therefore \frac{T_4}{T_1} = \left( \frac{T_3}{T_2} \right)^\gamma \text{ and } \frac{T_1}{T_2} = \left( \frac{V_2}{V_1} \right)^{\gamma^{-1}} = \frac{1}{\gamma^\gamma^{-1}}$$

Substituting in eq. 36, we get
Now \( \frac{T_3}{T_2} = \frac{V_3}{V_2} = r_c = \text{cut-off ratio} \)

\[
\eta = 1 - \frac{1}{r^{y-1}} \left[ \frac{r_c^y - 1}{\gamma(r_c - 1)} \right] \quad (38)
\]

When Eq. 38 is compared with Eq. 20, it is seen that the expressions are similar except for the term in the parentheses for the Diesel cycle. It can be shown that this term is always greater than unity.

Now \( r_e = \frac{V_3}{V_2} = \frac{V_3}{V_4} \frac{V_2}{V_1} = \frac{r}{r_e} \) where \( r \) is the compression ratio and \( r_e \) is the expansion ratio.

Thus, the thermal efficiency of the Diesel cycle can be written as

\[
\eta = 1 - \frac{1}{r^{y-1}} \left[ \frac{r_c^y r_e - 1}{\gamma(r_c - 1)} \right] \quad (39)
\]

Let \( r_c = r - \Delta \) since \( r \) is greater than \( r_e \). Here, \( \Delta \) is a small quantity. We therefore have

\[
\frac{r}{r_e} = \frac{r}{r - \Delta} = \frac{r}{r \left(1 - \frac{\Delta}{r} \right)} = \left(1 - \frac{\Delta}{r} \right)^{-1}
\]

We can expand the last term binomially so that

\[
\left(1 - \frac{\Delta}{r} \right)^{-1} = 1 + \frac{\Delta}{r} + \frac{\Delta^2}{r^2} + \frac{\Delta^3}{r^3} + \ldots
\]
Also \( \left( \frac{r}{r_e} \right)^\gamma = \left( \frac{r^\gamma}{(r-\Delta)^\gamma} \right) = \frac{r^\gamma}{r^\gamma \left( 1 - \frac{\Delta}{r} \right)^\gamma} = \left( 1 - \frac{\Delta}{r} \right)^{-\gamma} \)

We can expand the last term binomially so that
\[
\left( 1 - \frac{\Delta}{r} \right)^{-\gamma} = 1 + \gamma \frac{\Delta}{r} + \frac{\gamma(\gamma+1)\Delta^2}{2!} \frac{1}{r^2} + \frac{\gamma(\gamma+1)(\gamma+2)\Delta^3}{3!} \frac{1}{r^3} + \ldots
\]

Substituting in Eq. 39, we get
\[
\eta = 1 - \frac{1}{r^{\gamma-1}} \left[ \frac{\Delta}{r} + \frac{(\gamma+1)\Delta^2}{2!} \frac{1}{r^2} + \frac{(\gamma+1)(\gamma+2)\Delta^3}{3!} \frac{1}{r^3} + \ldots \right]
\]

Since the coefficients of \( \frac{\Delta}{r}, \frac{\Delta^2}{r^2}, \frac{\Delta^3}{r^3}, \) etc. are greater than unity, the quantity in the brackets in Eq. 40 will be greater than unity. Hence, for the Diesel cycle, we subtract \( \frac{1}{r^{\gamma-1}} \) times a quantity greater than unity from one, hence for the same \( r \), the Otto cycle efficiency is greater than that for a Diesel cycle.

If \( \frac{\Delta}{r} \) is small, the square, cube, etc of this quantity becomes progressively smaller, so the thermal efficiency of the Diesel cycle will tend towards that of the Otto cycle.

From the foregoing we can see the importance of cutting off the fuel supply early in the forward stroke, a condition which, because of the short time available and the high pressures involved, introduces practical difficulties with high speed engines and necessitates very rigid fuel injection gear.

In practice, the diesel engine shows a better efficiency than the Otto cycle engine because the compression of air alone in the former allows a greater compression ratio to be employed. With a mixture of fuel and air, as in practical Otto cycle engines, the maximum temperature developed by compression must not exceed the self ignition temperature of the mixture; hence a definite limit is imposed on the maximum value of the compression ratio.
Thus Otto cycle engines have compression ratios in the range of 7 to 12 while diesel cycle engines have compression ratios in the range of 16 to 22.

\[
\text{mep} = \frac{1}{V_s} \left[ P_2 (V_3 - V_2) + \frac{P_V P_3 - P_4 V_3 - P_2 V_2 - P_1 V_1}{V - 1} \right] \quad (42)
\]

The pressure ratio \( P_3/P_2 \) is known as explosion ratio \( r_p \)

\[
\frac{P_2}{P_1} = \left( \frac{V_1}{V_2} \right)^{\nu} \Rightarrow P_2 = P_1 r^{\nu},
\]

\[
P_3 = P_2 = P_1 r^{\nu},
\]

\[
P_4 = P_3 \left( \frac{V_3}{V_4} \right)^{\nu} = P_1 r^{\nu} \left( \frac{V_2}{V_1} \right)^{\nu} = P_1 r^{\nu},
\]

\[
V_4 = V_1, V_2 = V_c,
\]

\[
\frac{V_1}{V_2} = r,
\]

\[
\therefore \frac{V_c}{V_c} = r
\]

Substituting the above values in Eq 42 to get Eq (42A)

In terms of the cut-off ratio, we can obtain another expression for \( \text{mep}/p_1 \) as follows

\[
\text{mep} = P_1 \frac{\gamma \gamma (r_c - 1) - r (r_c - 1)}{(r - 1)(\gamma - 1)} \quad (42.A)
\]

We can obtain a value of \( r_c \) for a Diesel cycle in terms of \( Q' \) as follows:

\[
r_c = \frac{Q'}{c_p T_i r^{\gamma - 1}} + 1 \quad (41)
\]

We can substitute the value of \( \eta \) from Eq. 38 in Eq. 26, reproduced below and obtain the value of \( \text{mep}/p_1 \) for the Diesel cycle.
\[
\frac{mep}{p_1} = \eta \frac{Q'}{c_v T_1} \left[ 1 - \frac{1}{r} \right] (\gamma - 1)
\]

For the Diesel cycle, the expression for \( \text{mep}/p_3 \) is as follows:

\[
\frac{mep}{p_3} = \frac{mep}{p_1} \left( \frac{1}{r^\gamma} \right) \tag{43}
\]

Modern high speed diesel engines do not follow the Diesel cycle. The process of heat addition is partly at constant volume and partly at constant pressure. This brings us to the dual cycle.

1.7 The Dual Cycle

Process 2-3: Constant volume heat addition.
Process 3-4: Constant pressure heat addition.
Process 4-5: Reversible adiabatic expansion.
Process 5-1: Constant volume heat reject

An important characteristic of real cycles is the ratio of the mean effective pressure to the maximum pressure, since the mean effective pressure represents the useful (average) pressure acting on the piston while the maximum pressure represents the pressure which chiefly affects the strength required of the engine structure. In the constant-volume cycle, shown in Fig. 8, it is seen that the quantity $\text{mep}/p_3$ falls off rapidly as the compression ratio increases, which means that for a given mean effective pressure the maximum pressure rises rapidly as the compression ratio increases. For example, for a mean effective pressure of 7 bar and $Q'/c,T_1$ of 12, the maximum pressure at a compression ratio of 5 is 28 bar whereas at a compression ratio of 10, it rises to about 52 bar. Real cycles follow the same trend and it becomes a practical necessity to limit the maximum pressure when high compression ratios are used, as in diesel engines. This also indicates that diesel engines will have to be stronger (and hence heavier) because it has to withstand higher peak pressures.

Fig.9: T-S Diagram of Carnot Cycle.
Constant pressure heat addition achieves rather low peak pressures unless the compression ratio is quite high. In a real diesel engine, in order that combustion takes place at constant pressure, fuel has to be injected very late in the compression stroke (practically at the top dead center). But in order to increase the efficiency of the cycle, the fuel supply must be cut off early in the expansion stroke, both to give sufficient time for the fuel to burn and thereby increase combustion efficiency and reduce after burning but also reduce emissions. Such situations can be achieved if the engine was a slow speed type so that the piston would move sufficiently slowly for combustion to take place despite the late injection of the fuel. For modern high speed compression ignition engines it is not possible to achieve constant pressure combustion. Fuel is injected somewhat earlier in the compression stroke and has to go through the various stages of combustion. Thus it is seen that combustion is nearly at constant volume (like in a spark ignition engine). But the peak pressure is limited because of strength considerations so the rest of the heat addition is believed to take place at constant pressure in a cycle. This has led to the formulation of the dual combustion cycle. In this cycle, for high compression ratios, the peak pressure is not allowed to increase beyond a certain limit and to account for the total addition, the rest of the heat is assumed to be added at constant pressure. Hence the name limited pressure cycle.

The cycle is the equivalent air cycle for reciprocating high speed compression ignition engines. The P-V and T-s diagrams are shown in Figs.8 and 9. In the cycle, compression and expansion processes are isentropic; heat addition is partly at constant volume and partly at constant pressure while heat rejection is at constant volume as in the case of the Otto and Diesel cycles.

The heat supplied, \( Q_s \) per unit mass of charge is given by

\[
c_v(T_3 - T_2) + c_p(T_3' - T_2)
\]

whereas the heat rejected, \( Q_r \) per unit mass of charge is given by

\[
c_v(T_4 - T_1)
\]

and the thermal efficiency is given by
\[ \eta_{th} = 1 - \frac{c_v(T_4 - T_1)}{c_v(T_3' - T_2) + c_p(T_3' - T_2)} \quad (44A) \]

\[
1 - \frac{T_1 \left( \frac{T_4}{T_1} - 1 \right)}{T_2 \left( \frac{T_3}{T_2} - 1 \right) + \gamma T_3 \left( \frac{T_3'}{T_3} - 1 \right)}
\quad (44B) \]

\[
1 - \frac{T_4}{T_1} - 1
\frac{T_2 \left( \frac{T_3}{T_2} - 1 \right) + \gamma T_3 \left( \frac{T_3'}{T_3} - 1 \right)}{(44C)}
\]

From thermodynamics

\[
\frac{T_3}{T_2} = \frac{p_3}{p_2} = r_p \quad (45)
\]

the explosion or pressure ratio

and

\[
\frac{T_3'}{T_3} = \frac{V_3'}{V_3} = r_c \quad (46)
\]

the cut-off ratio.

Now, \( \frac{T_4}{T_1} = \frac{p_4}{p_1} = \frac{p_4}{p_3} \frac{p_3}{p_2} \frac{p_2}{p_1} \)

Also \( \frac{p_4}{p_3} = \left( \frac{V_3'}{V_4} \right)^\gamma = \left( \frac{V_3'}{V_3} \frac{V_4}{V_4} \right)^\gamma = \left( r_c \frac{1}{r} \right)^\gamma \)

And \( \frac{p_2}{p_1} = r^\gamma \)

Thus \( \frac{T_4}{T_1} = r_p r_c^\gamma \)

Also \( \frac{T_2}{T_1} = \left( \frac{V_1}{V_2} \right)^\gamma = r^{\gamma - 1} \)
Therefore, the thermal efficiency of the dual cycle is

\[
\eta = 1 - \frac{1}{r^{\gamma-1}} \left[ \frac{r_p r_c^{\gamma} - 1}{(r_p - 1) + \gamma r_p (r_c - 1)} \right]
\]  

(46)

We can substitute the value of \( \eta \) from Eq. 46 in Eq. 26 and obtain the value of \( \frac{m_{\text{ep}}}{p_1} \) for the dual cycle.

In terms of the cut-off ratio and pressure ratio, we can obtain another expression for \( \frac{m_{\text{ep}}}{p_1} \) as follows:

\[
\frac{m_{\text{ep}}}{p_1} = \gamma r_p (r_c - 1) + r^\gamma (r_p - 1) - r (r_p r_c^{\gamma} - 1)
\]

\[
\frac{m_{\text{ep}}}{p_1} = \frac{(r - 1)(\gamma - 1)}{(r - 1)(\gamma - 1)}
\]

(47)

For the dual cycle, the expression for \( \frac{m_{\text{ep}}}{p_3} \) is as follows:

\[
\frac{m_{\text{ep}}}{p_3} = m_{\text{ep}} \left( \frac{p_1}{p_3} \right)
\]

(48)

Since the dual cycle is also called the limited pressure cycle, the peak pressure, \( p_3 \), is usually specified. Since the initial pressure, \( p_1 \), is known, the ratio \( p_3/p_1 \) is known. We can correlate \( r_p \) with this ratio as follows:

\[
r_p = \frac{p_3}{p_1} \left( \frac{1}{r^{\gamma-1}} \right)
\]

(49)

We can obtain an expression for \( r_c \) in terms of \( Q' \) and \( r_p \) and other known quantities as follows:

\[
r_c = \frac{1}{\gamma} \left[ \left( \frac{Q'}{c_v T_1 r^{\gamma-1}} \right) \frac{1}{r_p} \right] + (\gamma - 1)
\]

(50)

We can also obtain an expression for \( r_p \) in terms of \( Q' \) and \( r_c \) and other known quantities as follows:
1.8 Stirling cycle

When a confined body of gas (air, helium, whatever) is heated, its pressure rises. This increased pressure can push on a piston and do work. The body of gas is then cooled, pressure drops, and the piston can return. The same cycle repeats over and over, using the same body of gas. That is all there is to it. No ignition, no carburetion, no valve train, no explosions. Many people have a hard time understanding the Stirling because it is so much simpler than conventional internal combustion engines.

The Stirling cycle is described using the pressure-volume (P-v) and temperature-entropy (T-s) diagrams shown in Figure 1. The P-v and T-s diagrams show the state of a "working fluid" at any point during the idealized cycle. The working fluid is normally a gas...in the Stirling engines being produced to us, the working fluid is air.

In the idealized Stirling cycle heat (i.e., energy) is transferred to the working fluid during the segment 2-3-4. Conversely, heat (energy) is extracted from the working fluid during the segment 4-1-2. During segment 2-3 heat is transferred to the fluid internally via regeneration of the energy transferred from the fluid during segment 4-1. The means that (ideally) heat is added from an external source only during segment 3-4, and that heat is rejected to the surrounding environment only during segment 1-2. Note that this is the idealized cycle.
Form 1 $\rightarrow$ to 2

1→2 It is an isothermal process, the piston in contact with cold reservoir is compressed isothermally, hence heat $|Q_c|$ has been rejected, and (isothermal compression $\rightarrow dU = 0$, W is positive and $Q_c$ is negative) the heat rejected is

$$Q_c = P_1V_1 \ln \frac{V_1}{V_2} = RT_c \ln(r_c)$$
2 → 3 It is an isochoric process, the left piston moves down while the right piston moves up. The volume of the system is kept constant, thus no work has been done by the system, but heat $Q_R$ has been input to the system by the regenerator which causes temperature to raise to $\theta_H$.

3 → 4 It is an isothermal expansion process, the left piston in contact with hot reservoir expanded isothermally at temperature $\theta_H$. Therefore

$$Q_H = P_3 V_3 \ln \frac{V_4}{V_3} = RT_H \ln(r_e)$$
4 → 1 It is an isochoric process which is a reversed process of 2 → 3, but from $\theta_H$ to $\theta_C$. The efficiencies of Stirling engine is

$$\eta = 1 - \frac{Q_c}{Q_H} = 1 - \frac{RT_c \ln(r_c)}{RT_H \ln(r_c)} = 1 - \frac{T_c}{T_H}$$

$$\eta_{\text{carnot}} = \eta_{\text{stirling}}$$

Consider regenerator efficiency $\eta_r$,

$$Q_H = RT_H \ln(r_c) + (1 - \eta_r)C_v(T_H - T_c)$$

$$Q_c = RT_c \ln(r_c) + (1 - \eta_r)C_v(T_H - T_c)$$

$$\eta_c = \frac{R \ln(r_c)(T_H - T_c)}{RT_c \ln(r_c) + (1 - \eta_r)C_v(T_H - T_c)} \quad \text{if}\quad \eta_r = 1$$

$$\eta_{st} = \frac{T_H - T_c}{T_H}$$

1.8 Comparison of Otto, Dual and Diesel cycles

In the previous articles we studied about Otto cycle, diesel cycle and dual cycle and looked at their thermal efficiency. In this article we will take a collective look at these three cycles in
order to compare and contrast them, so that we can come to know the relative advantages and disadvantages of these cycles.

1.8.1 Comparison based on same maximum pressure and heat rejection

In this article we will focus on peak pressure, peak temperature and heat rejection. The P-V and T-S diagrams of these three cycles for such a situation are drawn simultaneously as described below.

Figure 10 P-V and T-S diagram showing the comparison of Otto, Diesel and Dual cycles
In the above diagrams the following are the cycles

- Otto cycle: 1 – 2 – 3 – 4 – 1
- Dual cycle: 1 – 2’ – 3’ – 3 – 4 – 1
- Diesel cycle: 1 – 2” – 3 – 4 – 1

Remember that we are assuming the same peak pressure denoted by Pmax on the P-V diagram. And from the T-S diagram we know that T3 is the highest of the peak temperature which is again same for all three cycles under consideration. Heat rejection given by the area under 4 – 1 – 5 – 6 in the T-S diagram is also same for each case.

In this case the compression ratio is different for each cycle and can be found by dividing V1 with the respective V2 volumes of each cycle from the P-V diagram. The heat supplied or added in each cycle is given by the areas as follows from the T-S diagram

- Otto cycle: Area under 2 – 3 – 6 – 5 say q1
- Dual cycle: Area under 2’ – 2’ – 3 – 6 - 5 say q2
- Diesel cycle: Area under 2” – 3 – 6 – 5 say q3

It can also be seen from the same diagram that q3>q2>q1

We know that thermal efficiency is given by \( 1 - \text{heat rejected}/\text{heat supplied} \)

Since heat rejected is

Thermal efficiency of these engines under given circumstances is of the following order

**Diesel>Dual>Otto**

Hence in this case it is the diesel cycle which shows greater thermal efficiency.
1.8.2 **Comparison based on same compression ratio and heat rejection**

In this article we will focus on constant compression ratio and constant heat rejection. The P-V and T-S diagrams of these three cycles for such a situation are drawn simultaneously as described below.

![P-V and T-S diagram showing the comparison of Otto, Diesel and Dual cycles](image)

**Figure 20** P-V and T-S diagram showing the comparison of Otto, Diesel and Dual cycles
In the above diagrams the following are the cycles

- Otto cycle: 1 – 2 – 3 – 4 – 1
- Dual cycle: 1 – 2 – 2’ – 3’ – 4 – 1
- Diesel cycle: 1 – 2 – 3” – 4 – 1

Remember that we are assuming constant compression ratio for all three cycles which is given by V1/V2

The other parameter which is constant is the heat rejected from the cycle which is given by the following in each case as per the T-S diagram

**All cycles: Area under 4 – 1 – 5 – 6 in the T-S diagram**

The heat supplied is different in each case and can be established from the T-S diagram as follows

- Otto cycle: Area under 2 – 3 – 6 – 5 say q1
- Dual cycle: Area under 2 – 2’ – 3’ – 6 - 5 say q2
- Diesel cycle: Area under 2 – 3” – 6 – 5 say q3

It can also be seen from the same diagram that \( q_3 < q_2 < q_1 \)

We know that thermal efficiency is given by \( 1 - \text{heat rejected}/\text{heat supplied} \)

Since heat rejected is same and we know the order of magnitude of heat supplied, we can combine this information to conclude that

**Thermal efficiency**

**Otto>Dual>Diesel**

Hence we see that in this case as well the Otto cycle shows higher thermal efficiency than a dual cycle and even better than the diesel cycle.