GEOTECHNICAL ENGINEERING – II (Subject Code: 06CV64)  
UNIT 4: FLOW NETS

4.1 Introduction

In this chapter the topics that are covered include principles of seepage analysis, graphical solutions for seepage problems (flow nets), estimation of quantity of seepage and exit gradient, determination of phreatic line in earth dams with and without filter, piping effects and protective filter.

Objectives of this study are to understand basic principles of two dimensional flows through soil media. This understanding has application in the problems involving seepage flow through soil media and around impermeable boundaries which are frequently encountered in the design of engineering structures.

Two dimensional flow problems may be classified into two types namely,

1. Confined flow; for example, Flow of water through confined soil stratum.
2. Unconfined flow; for example, Flow of water through body of the earth dam.

These problems are to be addressed in geotechnical engineering in order to meet the following objectives of practical importance:

- To calculate quantity of flow (seepage) – incase of both confined & unconfined flow
- To obtain seepage pressure distribution and uplift pressures (stability analysis)
- To verify piping tendencies leading to instability

Preliminaries required for good understanding of this topic include continuity equation, Darcy’s law of permeability, and the validity, limitations and assumptions associated with Darcy’s law.

Water flows from a higher energy to a lower energy and behaves according to the principles of fluid mechanics. The sum of velocity head, pressure head and elevation head at any point constitute total head or hydraulic head which is causing the flow. According to Bernoulli’s energy principle, this total head responsible for flow of water is constant in any flow regime. However in case of flow through soil medium the velocity of flow is very small, hence contribution from velocity head is usually disregarded. The flow of water is governed by continuity equation. The continuity equation for steady state two dimensional flow with x and y velocity components \( v_x \) and \( v_y \) respectively is given by,

\[
\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0
\]  

(4.1)

According to Darcy’s law, discharge velocity of flow in a porous soil medium is proportional to hydraulic gradient that is, \( v \propto h \). If \( k_x \) and \( k_y \) are the coefficients of permeability of soil, \( i_x \) and \( i_y \) are the hydraulic gradients in \( x \) and \( y \) directions respectively and \( h(x, y) \) is the hydraulic head of flow, then

\[
v_x = k_x i_x = k_x \frac{\partial h}{\partial x}; \quad \text{and} \quad v_y = k_y i_y = k_y \frac{\partial h}{\partial y}
\]  

(4.2)
4.2 Seepage analysis – Laplace’s equation

Laplace’s equation governs the flow of an incompressible fluid, through an incompressible homogeneous soil medium. Combining continuity equation (Equation 4.1) and Darcy’s equations (Equation 4.2) yields,

\[ k_x \frac{\partial^2 h}{\partial x^2} + k_y \frac{\partial^2 h}{\partial y^2} = 0 \] (4.3)

For the case of isotropic soil the permeability coefficient is independent of direction that is, \( k_x = k_y = k \) thus Equation 4.3 becomes,

\[ \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0 \] (4.4)

This equation is known as the Laplace's equation. The two dimensional flow of water through soil is governed by Laplace’s equation. Laplace’s equation describes the energy loss associated with flow through a medium. Laplace’s equation is used to solve many kinds of flow problems, including those involving flow of heat, electricity, and seepage.

Assumptions associated with Laplace’s equation are stated below; the limitation implied by each of these assumptions is stated within the brackets.

1. Darcy’s law is valid 
   (Flow is laminar)
2. The soil is completely saturated 
   (Degree of saturation is 100%)
3. The soil is homogeneous 
   (Coefficient of permeability is constant everywhere in the soil medium)
4. The soil is isotropic 
   (Coefficient of permeability is same in all directions)
5. During flow, the volume of soil & water remains constant 
   (No expansion or contraction)
6. The soil and water are incompressible 
   (No volume change occurs)

Laplace equation is a partial differential equation. When Laplace equation (Equation 4.4) is solved graphically the equation gives flow net consisting two sets of curves intersecting at right angles known as flow lines (or stream lines) and equipotential lines.

4.3 Flow nets

The solution of Laplace equation requires knowledge of the boundary conditions. Geotechnical problems have complex boundary conditions for which it is difficult to obtain a closed form solution. Approximate methods such as graphical methods and numerical methods are often employed. Flow net technique is a graphical method, which satisfies Laplace equation. A flow net is a graphical representation of a flow field (Solution of Laplace equation) and comprises a family of flow lines and equipotential lines (Refer to Figure 4.1)
4.3.1 Characteristics of flow nets

1. Flow lines or stream lines represent flow paths of particles of water
2. Flow lines and equipotential line are orthogonal to each other
3. The area between two flow lines is called a flow channel
4. The rate of flow in a flow channel is constant (Δq)
5. Flow cannot occur across flow lines
6. An equipotential line is a line joining points with the same head
7. The velocity of flow is normal to the equipotential line
8. The difference in head between two equipotential lines is called the potential drop or head loss (Δh)
9. A flow line cannot intersect another flow line.
10. An equipotential line cannot intersect another equipotential line.

Figure 4.1: Flow net and its characteristics

4.3.2 Quantity of Seepage:

As mentioned earlier the main application of flow net is that it is employed in estimating quantity of seepage. If \( H \) is the net hydraulic head of flow, the quantity of seepage due to flow may estimated by drawing flow net part of which is shown in Figure 4.1. The flow net must be drawn by considering appropriate boundary conditions and adhering to characteristics of flow nets stated earlier in paragraph 4.3.1. With reference to Figure 4.1 following terms may be defined in order to estimate quantity of seepage.

\( N_d = \) Number of equipotential drops, that is, number of squares between two adjacent streamlines (Flow Lines) from the upstream equipotential to downstream equipotential.
\( N_f = \) Number of flow channels that is, number of squares between two adjacent equipotential lines from one boundary streamline to the other boundary streamline
\( \Delta q = \) flow through one flow channel (between two adjacent streamlines)
\( \Delta h = \) head loss between two adjacent equipotential lines
Consider a flow grid of dimension $a \times b$ (Figure 4.1)

Where,

$b = \Delta l$ – distance between equipotential lines &

$a = A$ – area across flow channel

Head loss for every potential drop: $\Delta h = \frac{H}{N_d}$ &

Hydraulic gradient: $i = \frac{\Delta h}{\Delta l} = \frac{\Delta h}{b}$

But, $\Delta h = \frac{H}{N_d}$, $i = \frac{H}{b N_d}$

Flow per channel (Darcy’s law): $\Delta q = v i = k i A = k \frac{H}{b N_d} a$

Total flow per unit width across each flow channel: $q = \Delta q N_f = k \frac{H}{b N_d} a$

If $a = b$, then, **Total flow/unit width** $q = k \frac{H}{b} \left( \frac{N_f}{N_d} \right) a$

($a = b$, implies that each flow grid is square)

Thus the quantity of seepage across total width $L$ of soil medium beneath the dam is given by

$$Q = q \times L = \left[ k \frac{H}{b} \left( \frac{N_f}{N_d} \right) \right] \times L \quad (4.5)$$

Where,

$N_d =$ Number of equipotential drops, $N_f =$ Number of flow channels,

$k =$ coefficient of permeability & $H =$ Net hydraulic head

**4.3.3 Guidelines for drawing flow nets**

- Draw to a convenient scale the cross sections of the structure, water elevations, and soil deposit profiles.
- Establish boundary conditions that is, Identify impermeable and permeable boundaries. The soil and impermeable boundary interfaces are flow lines. The soil and permeable boundary interfaces are equipotential lines.
- Draw one or two flow lines and equipotential lines near the boundaries. Sketch intermediate flow lines and equipotential lines by smooth curves adhering to right-angle intersections such that area between a pair of flow lines and a pair of equipotential lines is approximately a curvilinear square grid.
- Where flow direction is a straight line, flow lines are equal distance apart and parallel. Also, the flownet in confined areas between parallel boundaries usually consists of flow lines and equipotential lines that are elliptical in shape and symmetrical.
Try to avoid making sharp transition between straight and curved sections of flow and equipotential lines. Transitions must be gradual and smooth. Continue sketching until a problem develops.

Successive trials will result in a reasonably consistent flow net. In most cases, 3 to 8 flow lines are usually sufficient. Depending on the number of flow lines selected, the number of equipotential lines will automatically be fixed by geometry and grid layout.

### 4.3.4 Typical illustrations of flownet

Following illustrations, Figures 4.2, Figures 4.3, Figures 4.4 and Figures 4.5 demonstrate the typical flow nets drawn for different kinds of seepage problems pertaining to flow beneath hydraulic structures like, dam, sheet pile, dam with sheet pile as heel cutoff wall and dam with sheet pile as toe cutoff wall respectively.

**Figure 4.2:** Typical flow net for the flow beneath the dam without any cutoff wall
[Lambe & R.V. Whitman (1979)]

**Figure 4.3:** Typical flow net for the flow around a sheet pile wall
4.4 Exit gradient

**Hydraulic Gradient:** The potential drop between two adjacent equipotential lines divided by distance between them is known as hydraulic gradient. Thus, the hydraulic gradient across any square in the flow net involves measuring the average dimension of the square. The maximum value of hydraulic gradient which results in maximum seepage velocity occurs across smallest square (flow grid).

**Exit Gradient:** The exit gradient is the hydraulic gradient at the downstream end of the flow line where percolating water leaves the soil mass and emerges into the free water at the downstream.
4.4.1 Maximum Exit Gradient: dams and sheet pile walls

The maximum exit gradient for the cases of both *dams and sheet pile walls* can be determined from the flow net. The maximum exit gradient is given by

\[ i_{exit} = \frac{\Delta h}{l} \]  

(4.6)

Where, \( \Delta h \) is the head lost between the last two equipotential lines, and the \( l \) length of the flow element (Refer to Figure 4.6)

![Figure 4.6: Computation of maximum exit gradient](image)

![Figure 4.7: Computation of maximum exit gradient for sheet pile wall](image)
### 4.4.2 Maximum Exit Gradient: Sheet pile walls

Maximum exit gradient Sheet pile walls can be computed alternatively as explained below. A theoretical solution for the determination of the maximum exit gradient for a single row of sheet pile structures is available and is of the form (Refer to Figure 4.7)

\[
i_{exit} = \frac{1}{\pi} \left[ \frac{\text{Maximum hydraulic head}}{\text{depth of penetration of sheet pile}} \right] = \frac{1}{\pi} \left[ \frac{H_1 - H_2}{l_d} \right]
\]  

(4.7)

### 4.4.3 Critical Hydraulic Gradient

Consider a case of water flowing under a hydraulic head \(x\) through a soil column of height \(H\) as shown in the Figure 4.8.

![Figure 4.8: Computation of critical hydraulic gradient at point O.](image)

The state of stress at point O situated at a depth of \(h_2\) from the top of soil column may be computed as follows,

Vertical stress at O is,

\[\sigma_{y_o} = h_1 \gamma_w + h_2 \gamma_{sat}\]

If \(\gamma_w\) is the unit weight of water then pore pressure \(u_o\) at O is,

\[u_o = (h_1 + h_2 + x) \gamma_w\]
If \( \gamma_{sat} \) and \( \gamma \) saturated and submerged unit weights of the soil column respectively,

Then effective stress at O is,

\[
\sigma' = \sigma_0 - u_o = (h_1 \gamma_w + h_2 \gamma_{sat}) - (h_1 + h_2 + x) \gamma_w \\
\sigma' = h_2 (\gamma_{sat} - \gamma_w) - x \gamma_w = h_2 \gamma' - x \gamma_w \\
\]

For quick sand condition (sand boiling) the effective stress tends to zero; that is, \( \sigma' = 0 \)

We get critical hydraulic gradient \( i_{critical} \) as,

\[
\sigma' = h_2 \gamma' - x \gamma_w = 0; \\
\]

\[
i_{critical} = \frac{x \gamma_w}{h_2 \gamma_w} = \frac{\gamma_w (G - 1)}{1 + e} \times \frac{1}{\gamma_w} = \frac{G - 1}{1 + e} \\
\]

Where \( G \) is the specific gravity of the soil particles and \( e \) is the void ratio of the soil mass. Therefore critical hydraulic gradient corresponds to hydraulic gradient which tends to a state of zero effective stress. Hence critical hydraulic gradient is given by

\[
i_{critical} = \frac{G - 1}{1 + e} \quad (4.8)
\]

**4.5 Piping Effects**

Soils can be eroded by flowing water. Erosion can occur underground, beneath the hydraulic structures, if there are cavities, cracks in rock, or high exit gradient induced instability at toe of the dam, such that soil particles can be washed into them and transported away by high velocity seeping water. This type of underground erosion progresses and creates an open path for flow of water; it is called “piping”. Preventing piping is a prime consideration in the design of safe dams. Briefly the processes associated with initiation of piping in dams are as follows,

- Upward seepage at the toe of the dam on the downstream side causes local instability of soil in that region leading erosion.
- A process of gradual erosion and undermining of the dam may begin, this type of failure known as piping, has been a common cause for the total failure of earth dams
- The initiation of piping starts when exit hydraulic gradient of upward flow is close to critical hydraulic gradient

Factor of safety against piping is defined as,

\[
FS = \frac{i_{critical}}{i_{exit}} \\
\]

Where \( i_{exit} \) is the maximum exit gradient and \( i_{critical} \) is the critical hydraulic gradient (Equation 4.8). The maximum exit gradient can be determined from the flow net. A factor of safety of 3 to 4 is considered adequate for the safe performance of the structure against piping failure.
4.6 Design of Filters

In order to avoid failures of hydraulic structures due to piping effects, many remedial measures are available. Some of these remedial measures that are usually adopted in practice are shown in Figure 4.9. They include providing impervious blanket on the upstream and gravel filter at the toe as shown for the case of an earth dam (Figure 4.9). Main objectives of these measures are preventing buildup of high seepage pressure and migration of eroded soil particles.

![Figure 4.9: Remedial measures against piping](image)

When seepage water flows from a soil with relatively fine grains into a coarser material there is a danger that the fine soil particles may wash away into the coarse material. Over a period of time, this process may clog the void spaces in the coarser material. Such a situation can be prevented by the use of a protective filter between the two soils.

Conditions for the proper selection of the filter material are,

1. The size of the voids in the filter material should be small enough to hold the larger particles of the protected material in place.
2. The filter material should have a high permeability to prevent buildup of large seepage forces and hydrostatic pressures in the filters.

The experimental investigation of protective filters provided the following criteria which are to be followed to satisfy the above conditions:

**Criteria-1**

\[
\frac{D_{15(F)}}{D_{85(S)}} \leq 4 \text{ to } 5 \quad \text{----------------- To satisfy condition 1}
\]

**Criteria-2**

\[
\frac{D_{15(F)}}{D_{15(S)}} \geq 4 \text{ to } 5 \quad \text{----------------- To satisfy condition 2}
\]

Where,

- \(D_{15(F)}\) = diameter through which 15% of filter material will pass
- \(D_{15(S)}\) = diameter through which 15% of soil to be protected will pass
- \(D_{85(S)}\) = diameter through which 85% of soil to be protected will pass
4.7 Seepage Flow through Homogeneous Earth Dams

In order to draw flow net to find quantity of seepage through the body of the earth dam it is essential to locate top line of seepage. This upper boundary is a free water surface and will be referred to as the *line of seepage or phreatic line*. The seepage line may therefore be defined as the line above which there is no hydrostatic pressure and below which there is hydrostatic pressure. Therefore phreatic line is the top flow line which separates saturated and unsaturated zones within the body of the earth dam.

Therefore the problem of computation of the seepage loss through an earth dam primarily involves prediction of the position of the line of seepage in the cross-section.

4.7.1 Locating Phreatic Line

It has been noticed from experiments on homogeneous earth dam models that the line of seepage assumes more or less the shape of a parabola. Also, assuming that hydraulic gradient \( i \) is equal to the slope of the free surface and is constant with depth (Dupit’s theory), the resulting solution of the phreatic surface is parabola. In some sections a little divergence from a regular parabola is required particularly at the surfaces of entry and discharge of the line of seepage. The properties of the regular parabola which are essential to obtain phreatic line are depicted in Figure 4.10.

![Figure 4.10: geometrical properties of regular parabola](image)

Every point on the parabola is equidistant from focus and directrix.

Therefore,

\[
FA = AB
\]

Also,

\[
FG = GE = p = \frac{S}{2}
\]

Focus = (0,0)

Any point, \( A \) on the parabola is given by,

\[
A = A (x, z)\]

\[
x^2 + z^2 = (2p + x)^2\]

that is,

\[
x = \frac{z^2 - 4p^2}{4p}
\]
4.7.2 Phreatic line for an earth dam without toe filter

In the case of a homogeneous earth dam resting on an impervious foundation with no drainage filter, the top flow line ends at some point on the downstream face of the dam; the focus of the base parabola in this case happens to be the downstream toe of the dam itself as shown in Figure 4.11.

![Figure 4.11: Phreatic line for an earth dam without toe filter](image)

The following are the steps in the graphical determination of the top flow line for a homogeneous dam resting on an impervious foundation without filters:

1. Draw the earth dam section and upstream water level \(H\) to some convenient scale. Let Point-2 is the point on the upstream face coinciding with water level.

2. Let \(\Delta\) be the horizontal distance between point-2 and upstream heel of the dam. Locate Point-1 at a distance of 0.3 times \(\Delta\) from Point-2 on the water surface. That is distance 1-2 is 0.3\(\Delta\).

3. Focus of the base parabola is located at the downstream toe of the dam, that is Point-0 (distance 0-1 is \(d\)). Select \(x\)-\(z\) reference axis with focus 0 as origin.

4. Directrix of the parabola is at distance \(2p\) from the focus 0, where \(p\) is given by,

\[
p = \frac{1}{2} \left( \sqrt{d^2 + H^2} - d \right)
\]

5. By choosing suitable values of \(z\)-ordinates (for example; 0.2\(H\), 0.4\(H\) … & \(H\)) compute the \(x\)-ordinates of the base parabola using the relation,

\[
x = \frac{z^2 - 4p^2}{4p}.
\]

6. Join all these points to get base parabola starting from Point-1 and concluding at a point midway between focus-0 and directrix. This parabola will be correct for the central portion of the top flow line. Necessary corrections at the entry on the upstream side and at exist on the downstream side are to be made.

7. Upstream correction: The portion of the top flow line at entry is sketched visually to meet the boundary condition there that is phreatic line meets perpendicularly.
with the upstream face, which is a boundary equipotential and the phreatic line is made to meet the base parabola tangentially at a convenient point.

8. Downstream correction (Casagrande’s method): The breakout point on the downstream discharge face may be determined by measuring out \( L \) from the toe along the face. If \( \beta \) is the downstream slope angle then \( L \) may be computed from the following equations,

\[
\begin{align*}
\text{For } \beta < 30^\circ, & \quad L = \frac{d}{\cos \beta} - \frac{d^2}{\cos^2 \beta} - \frac{H^2}{\sin^2 \beta}; \\
\text{For } 30^\circ < \beta < 90^\circ, & \quad L = \sqrt{H^2 + d^2} - \sqrt{d^2 - H^2 \cot^2 \beta}
\end{align*}
\]

9. Finally the quantity of seepage flow through may be computed from the following equations,

\[
\begin{align*}
\text{For } \beta < 30^\circ, & \quad q = kL \sin \beta \tan \beta \\
\text{For } 30^\circ < \beta < 90^\circ, & \quad q = kL \sin^2 \beta
\end{align*}
\]

Where, \( k \) is the coefficient of permeability of the dam material.

### 4.7.3 Phreatic line for an earth dam with toe filter

The following are the steps in the graphical determination of the top flow line for a homogeneous dam with a toe filter: [Refer to Figures 4.12 (a), (b), (c), (d), & (e)]

1. Draw the earth dam section and upstream water level (\( H \)) to some convenient scale.
2. Locate Point-\( B \), the point on the upstream slope coinciding with water level.
3. Let \( \Delta \) be the horizontal distance between point- \( B \) and upstream heel of the dam. Locate Point-\( A \) at a distance of 0.3 times \( \Delta \) from Point-\( B \) on the water surface. That is distance \( AB \) is 0.3\( \Delta \) [Refer to Figure 4.12(a)]
4. Select \( F \) as the focus of the parabolic phreatic line, Point-\( F \) is located at the intersection of the bottom flow line and the downstream toe filter. Let horizontal distance between points \( A \) & \( F \) be \( d \) i.e., \( AF = d \)
5. Locate Point-\( G \) on the directrix of the parabola, located a distance 2\( P \) from the focal point, Point-\( F \), that is \( FG = 2P \) where, [Refer to Figure 4.12(b)]

\[
p = \frac{1}{2} \left( \sqrt{d^2 + H^2} - d \right)
\]
6. Select base of the dam and directrix as \( X \) & \( Z \) axes
Figure 4.12: Phreatic line for an earth dam with toe filter
7. By choosing suitable values of \( z \)-ordinates (for example; \( 0.2H, 0.4H \ldots \) & \( H \)) compute the \( x \)-ordinates of the base parabola using the relation, 
\[ x = \frac{z^2 - 4p^2}{4p}. \]
Thus \( z_1, z_2, z_3, z_4 \ldots \ldots \) are computed for the ordinates \( x_1, x_2, x_3, x_4 \ldots \ldots \) respectively [Refer to Figure 4.12(c)].

8. Join all such located points to get basic parabola. This parabola meets toe filter (equipotential line) orthogonally at midpoint of \( FG \) that is at a distance \( p \) from \( F \) (vertex \( K \) of the parabola). Joint points \( K-0-1-2-3-4-A \) to get parabola \( ABK \) [Refer to Figure 4.12(d)].

9. Apply modification to phreatic line at the entry Point-B on the upstream slope which is an equipotential line. Draw line perpendicular to upstream slope starting from \( B \) and meets the base parabola smoothly and tangentially at a convenient point say, \( C \). Complete the phreatic line \( BCK \) (top flow line) by joining \( BC \) erase remaining portion of the base parabola [Refer to Figure 4.12(e)].

10. Finally the quantity of seepage flow through may be compute from the following equations, 
Let the distance between \( F \) & \( G \) is \( S \)
i.e., the distance between focus and directrix, \( \therefore S = 2\times p \)
Then the quantity of seepage through unit length of dam is, 
\[ q = kS = k(2\times p) = 2kp \]
Where, \( k \) is the coefficient of permeability of the dam material.

### 4.8 Problems

#### Problem 4.1: (July/August – 2002)

For a homogeneous earth dam of 52 m height and 2 m free board, the flownet has 22 potential drops and 5 flow channels. Calculate the discharge per meter length of the dam, given \( k = 22 \times 10^{-6} \text{ m/sec} \)

**Solution: Problem 4.1**

*Data given:*

\( H = 52 - 2 = 50 \text{ m}, \ N_d = 22, \ N_f = 5, \ k = 22 \times 10^{-6} \text{ m/sec} \)

**Solution:**

\[ Q = k \times H \times \left( \frac{N_f}{N_d} \right) = 22 \times 10^{-6} \times 50 \times \frac{5}{22} \]

\[ Q = 2.5 \times 10^{-4} \frac{m^3}{s \times m} \text{ (Answer)} \]
Problem: 4.2

A concrete dam (Refer to figure P-4.1) 17.5 m base retains water to a level of 11.0 m on the upstream. The water level on the downstream is 2.0 m. the impervious stratum is 10.0 m below the dam. The coefficient of permeability \( k = 10^{-6} \) m/sec. If dam is 50 m long Compute total quantity of seepage flow per day below the dam. Also compute seepage pressure at point P, 5 m below the center of the dam.

Figure: P-4.1

Solution: Problem 4.2

Figure: P-4.2

- Draw the dam section to scale also locate point-P at which seepage pressure is required as shown in Figure P-4.2 (water levels need not be drawn to scale).
- Identify that \( AB \) & \( EF \) are equipotential lines and \( BC, CD, DE \) & \( GH \) are flow lines.
- Sketch flow lines and equipotential lines according to guidelines stated earlier (Section: 4.3.3). Draw two flow lines perpendicular to boundary equipotential lines and parallel to boundary flow lines identified in the previous step.
Complete the flow net by drawing sufficient number of equipotential lines such that they are perpendicular to flow lines and approximately square flow elements are formed. In this case nine such equipotential lines are drawn in order to satisfy these conditions as shown in Figure 4.3.

Thus three flow channels and ten equipotential lines are formed as illustrated in Figure P-4.3. Therefore \( N_f = 3 \) & \( N_d = 10 \).

Head drop for every equipotential drop is \( \Delta h = \left( \frac{H}{N_d} \right) = \left( \frac{9.0}{10} \right) = 0.90 \)

Seepage loss under the dam:

\( \text{Head} = H = 11 - 2 = 9.0 \text{ m} \) & \( k = 1 \times 10^6 \text{ m/s} \)

Therefore the discharge due to seepage is

\[
q = k \cdot H \cdot \left( \frac{N_l}{N_d} \right) = 10^{-6} \times 9 \times \frac{3}{10} = 2 \times 10^{-4} \text{ m}^3 \text{ per meter length}
\]

Therefore total discharge is,

\[ Q = 2 \times 10^{-4} \times 3600 \times 24 \times 50 = 864.0 \text{ m}^3 / \text{day} \]

Seepage pressure at \( P \): In the Figure P-4.3 identify that location of point \( P \) is coinciding with fifth equipotential line, i.e., there are 5 equipotential drops till point \( P \). seepage pressure at this location can be computed by using any arbitrary datum. For illustration purpose seepage pressure at \( P \) is computed for two different datum yielding same results.
Case -1:  
Datum is top of tail water
Total head at P is,
\((h_i)_p = 9.0 - \Delta h(5)\)
\((h_i)_p = 9.0 - \left( \frac{9}{10} \right)(5) = 4.5 \text{ m}\)
Elevation head, \((h_e)_p = -9.0 \text{ m}\)
Seepage pressure head,
\((h_s)_p = (h_i)_p - (h_e)_p\)
\((h_s)_p = 4.5 - (-9.0) = 13.5 \text{ m}\)
Pore pressure at P,
\(u_p = (h_s)_p \times \gamma_w = 13.5 \times 9.81\)
\(u_p = 132.435 \text{ kN} / \text{m}^2\)

Case -2:  
Datum is top of impervious layer
Total head at P is,
\((h_i)_p = 21.0 - \Delta h(5)\)
\((h_i)_p = 21.0 - \left( \frac{9}{10} \right)(5) = 16.5 \text{ m}\)
Elevation head, \((h_e)_p = +3.0 \text{ m}\)
Seepage pressure head,
\((h_s)_p = (h_i)_p - (h_e)_p\)
\((h_s)_p = 16.5 - (+3.0) = 13.5 \text{ m}\)
Pore pressure at P,
\(u_p = (h_s)_p \times \gamma_w = 13.5 \times 9.81\)
\(u_p = 132.435 \text{ kN} / \text{m}^2\)

Thus it can be concluded that seepage pressure can be computed considering any arbitrary datum to get same results. Hence seepage pressure head at P is, \((h_s)_p = 13.5 \text{ m}\) which yields the pore pressure at P as, \(u_p = 132.435 \text{ kN} / \text{m}^2\) (kPa).

Problem: 4.3 (July – 2007)

A soil stratum with permeability, \(k = 5 \times 10^{-7} \text{ cm/sec}\) overlies an impermeable stratum. The impermeable stratum lies at a depth of 18 m below the ground surface. A sheet pile wall penetrates 8 m into the permeable soil stratum. Water stands to a height of 9 m on upstream side and 1.5 m on down stream side, above the surface of soil stratum. Sketch the flownet and determine:

a. Quantity of seepage
b. The seepage pressure at a point P located 8 m below surface of soil stratum and 4 m away from the sheet pile wall on its upstream side
c. The pore water pressure at point P and
d. The maximum exit gradient

Solution: Problem 4.3

- Draw the dam section to scale also locate point-P at which seepage pressure is required as shown in Figure P-4.2 (water levels need not be drawn to scale). Identify the boundary conditions, i.e., boundary lines that represent equipotential lines and flow lines as shown in Figure 4.3.
- Sketch flow lines and equipotential lines according to guidelines stated earlier (Section: 4.3.3). Draw three flow lines perpendicular to boundary equipotential lines and parallel to boundary flow lines identified in the previous step.
Complete the flow net by drawing sufficient number of equipotential lines such that they are perpendicular to flow lines and approximately square flow elements are formed. In this case seven such equipotential lines are drawn in order to satisfy these conditions as shown in Figure 4.5.

Thus four flow channels and eight equipotential lines are formed as illustrated in Figure P-4.5. Therefore \( N_f = 4 \) & \( N_d = 8 \). Number of equipotential drops at the location of Point-\( P \) is 2.5 (\( P \) is located midway between 2\(^{nd} \) and 3\(^{rd} \) equipotential lines) i.e., \( (N_d)_p = 2.5 \).

Head drop for every equipotential drop is \( \Delta h = \left( \frac{7.5}{8} \right) = 0.9375 \).
Seepage loss under the sheet pile

Data: Head, \( H = 9.0 - 1.5 = 7.5 \) m,

Coeff. of permeability, \( k = 5 \times 10^{-7} \text{cm/s} = 5 \times 10^{-9} \text{m/s} \)

Therefore seepage loss is,

\[ Q = k \cdot H \left( \frac{N_f}{N_d} \right) = 5 \times 10^{-9} \times 7.5 \times \frac{4}{8} \]

\[ Q = 1.875 \times 10^{-8} \frac{m^3}{s \cdot m} \]

\[ Q = 1.875 \times 10^{-8} \times 3600 \times 24 = 1.62 \times 10^{-3} \text{ m}^3 / \text{day} / \text{m} \]

Seepage pressure & Pore pressure at P: In the Figure P-4.5 identify that location of point \( P \) is midway between 2\(^{nd} \) and 3\(^{rd} \) equipotential lines, i.e., there are 2.5 equipotential drops till point \( P \). Seepage pressure at this location can be computed by using any arbitrary datum. For illustration purpose seepage pressure at \( P \) is computed for two different datum yielding same results.

### Case - 1:

**Datum is top of tail water**

Total head at \( P \) is,

\( (h_i)_P = 7.5 - \Delta h(2.5) \)

\( (h_i)_P = 7.5 - \left( \frac{7.5}{8} \right)(2.5) = 5.15625 \text{ m} \)

Elevation head, \( (h_e)_P = -9.50 \text{ m} \)

Seepage pressure head,

\( (h_s)_P = (h_i)_P - (h_e)_P \)

\( (h_s)_P = 5.15625 - (-9.50) = 14.65625 \text{ m} \)

Pore pressure at \( P \),

\( u_p = (h_s)_P \times \gamma_w = 14.65625 \times 9.81 \)

\( u_p = 143.7778 \text{ kN/m}^2 \)

### Case - 2:

**Datum is top of impervious layer**

Total head at \( P \) is,

\( (h_i)_P = 27.0 - \Delta h(2.5) \)

\( (h_i)_P = 27.0 - \left( \frac{7.5}{8} \right)(2.5) = 24.65625 \text{ m} \)

Elevation head, \( (h_e)_P = +10.0 \text{ m} \)

Seepage pressure head,

\( (h_s)_P = (h_i)_P - (h_e)_P \)

\( (h_s)_P = 24.65625 - (+10.0) = 14.65625 \text{ m} \)

Pore pressure at \( P \),

\( u_p = (h_s)_P \times \gamma_w = 14.65625 \times 9.81 \)

\( u_p = 143.7778 \text{ kN/m}^2 \)

Thus it can be concluded that seepage pressure can be computed considering any arbitrary datum to get same results. Hence seepage pressure head at \( P \) is, \( (h_s)_P = 14.656 \text{ m} \) which yields the pore pressure at \( P \) as, \( u_p = 143.7778 \text{ kN/m}^2 \) (kPa)

**Maximum exit gradient:**

Maximum exit gradient for flow beneath the sheet pile wall may be computed using any of the two methods explained 4.4.1 and 4.4.2. Herein both of these methods are employed to compute maximum exit gradient. In case of first method dimension of the last flow
element is used (Figure P-4.6), while in the second method depth of penetration of sheet pile is used to compute maximum exit gradient.

**Method – 1**

The head lost between the last two equipotential lines $\Delta h$, and the $l$ length of the last flow element adjacent to sheet pile is used to compute the exit gradient. The length $l$ of the last element, as shown in Figure P-4.6, is equal to 3.1 m. Therefore, using equation 4.6, we get

$$\Delta h = \frac{H_1 - H_2}{N_d} = \frac{9.0 - 1.5}{8} = 0.9375$$

$$i_{exit} = \frac{\Delta h}{l} = \frac{0.9375}{3.1} = 0.3024$$

**Maximum exit gradient = 0.3024**

**Method – 2**

The maximum exit gradient for a single row of sheet pile structures is computed using depth of embedment of sheet pile, $l_d$. For the present case, $l_d = 8$ m. Therefore, using equation 4.7, we get

$$i_{exit} = \frac{1}{\pi} \left[ \frac{H_1 - H_2}{l_d} \right] = \frac{1}{\pi} \left[ \frac{9.0 - 1.5}{8} \right] = 0.2984$$

**Maximum exit gradient = 0.2984**

*Note: The maximum exit gradient computed using two methods closely agrees with a difference of about 1.32 % in the result.*
An earth dam has the following details: Top width 10 m., upstream and downstream slopes 2.5 horizontal to 1 vertical. Total height of dam 35 m., the height of water stored 30 m. Downstream filter 60 m. long. K for dam material 3.75x10^{-3} mm/sec. Draw the phreatic line and calculate the discharge through the dam.

Solution: Problem 4.4

For the present problem, the solution is obtained by referring to the general diagram of phreatic line of the dam shown in Figure P-4.7

![Figure P-4.7](image)

Referring to Figure P-4.7 and to the procedure described in section 4.7.3, the data given in the present problem are interpreted to obtain solution as follows,

Top width of the dam = ML = 10 m, Height of the dam = 35 m,

Height of water = BE = 30 m, Downstream filter length = FJ = 60.0 m

Upstream and downstream slope angles = \( \alpha = \beta = \tan^{-1}\left(\frac{1}{2.5}\right) = 21.80^\circ \)

Base width of the dam = \( DJ = ML + 35\times\frac{1}{\tan(\alpha)} = 10 + 35\times\frac{1}{21.802.5} = 185.0 \) m

\( DE = \Delta = BE\times\frac{1}{\tan(\alpha)} = 30\times2.5 = 75.0 \) m, \( AB = 0.3\Delta = 0.3\times75 = 22.5 \) m

With respect to origin F, coordinates of point \( A(x, z) = A(d, H) \) are,

(F is the focus of the base parabola)

\( x = d = DJ - (FJ + DE) + AB = 185.0 - (60.0 + 75.0) + 22.5 \)

\( x = 72.5 \) m \& \( z = H = 30.0 \) m, \( A(x, z) = (72.5, 30.0) \)

\( p = \frac{S}{2} = \frac{FG}{2}; \quad p = \frac{1}{2}\left(\sqrt{d^2 + H^2} - d\right) = \frac{1}{2}\left(\sqrt{72.5^2 + 30^2} - 72.5\right) \)

\( p = 2.981 \) m \( ; \quad S = 2\times p = 2\times2.981 = 5.962 \) m

Seepage loss through the dam is, \( q = k\times S \)
Where, coefficient of permeability \( k = 3.75 \times 10^{-3} \text{ mm/sec} \)

\[
\therefore q = 3.75 \times 10^{-6} \times 5.962 = 2.23575 \times 10^{-5} \text{ m}^3/\text{sec}
\]

\[
q = 2.23575 \times 10^{-5} \times 3600 \times 24 = 1.932 \text{ m}^3/\text{day/meter length of dam}
\]

**Note:** After point-A (starting point of base parabola) and directrix passing through point-G are fixed select \(x-z\) reference axes with \(F\) as origin of the base parabola. Obtain the coordinates \((x, z)\) of various points describing the base parabola using the value of \(p\) in the relation, \(x = \frac{z^2 - 4p^2}{4p}\). Join all points to get base parabola and draw phreatic line by applying appropriate correction on upstream face at point-B as shown in Figure P-4.7. The procedure is described in Section 4.7.3. This is left to the student as an exercise.

**Problem: 4.5**

An earth dam section is shown in Figure P-4.8 Determine the rate of seepage through the earth dam Assume that \(k = 10^{-5}\) m/min.

**Solution: Problem 4.5**

The given problem is concerned with seepage through dam without downstream filter. The general diagram of phreatic line of seepage flow is shown in Figure P-4.9. The detail procedure for plotting this phreatic line is described in Section 4.7.2.
Referring to Figure: P-4.9, following calculations are carried out using the procedure of Section 4.7.2.

Top width of the dam = 7.0 m, Height of the dam = 35 m, Height of water = 32 m

Upstream and downstream slope angles = \( \alpha = \beta = \tan^{-1}\left(\frac{1}{2.0}\right) = 26.565^\circ \)

Base width of the dam = \( 7.0 + \left(35 \times \left[\frac{1}{\tan(\alpha)} + \frac{1}{\tan(\beta)}\right]\right) = 7 + \left(35 \times \left[2 + 2\right]\right) = 147.0 \) m

\[ \Delta = 32 \times \frac{1}{\tan(\alpha)} = 32 \times 2 = 64.0 \text{ m}, \quad : \quad a\alpha = 0.3\Delta = 0.3 \times 64 = 19.2 \text{ m} \]

With respect to origin, \( a'(x, z) = a'(d, H) \) are, (Origin is focus of the base parabola)

\( x = d = 147.0 - \Delta + aa' = 147.0 - 64.0 + 19.2 = 102.2 \) m \& \( z = H = 32.0 \) m,

\( : \quad a'(x, z) = (102.2, 32.0) \), Using Casagrande's method \( (For \beta < 30^\circ) \), we have,

\[ L = bc = \frac{d}{\cos\beta} \cdot \sqrt{\frac{d^2}{\cos^2\beta} \cdot \frac{H^2}{\sin^2\beta}} = \frac{102.2}{\cos26.565} \cdot \sqrt{\frac{102.2^2}{\cos^226.565} \cdot \frac{32^2}{\sin^226.565}} = 25.18 \text{ m} \]

For \( \beta < 30^\circ \), Seepage loss through the dam is, \( q = kL \sin\beta \tan\beta \)

Where, coefficient of permeability \( k = 10^{-5} \text{ m / min} \)

\( : \quad q = 1 \times 10^{-5} \times 25.18 \times \sin26.565 \tan26.565 = 5.63 \times 10^{-5} \text{ m}^3 / \text{ min} \)

\[ q = 2.23575 \times 10^{-5} \times 60 \times 24 = 0.0811 \text{ m}^3 / \text{ day / meter length of dam} \]

Reference: